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General equilibrium models with Morishima elasticities of substitution in production☆

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ABSTRACT

Analytical general equilibrium (AGE) models are important tools that economists use to answer questions about theory and policy. When a production function has three or more inputs, the traditional modeling technique employs Allen elasticities of substitution to represent general functional forms. This paper builds an analytical general equilibrium model using the Morishima elasticity of substitution (MES). Specifically, an existing model using Allen elasticities is reformulated to employ the MES and the new, closed-form solutions are interpreted with additional insights from the reformulation. Importantly, the special case of constant elasticity of substitution (CES) production follows directly when using Morishima elasticities, but not Allen elasticities. This paper also provides a general technique for switching from Allen to Morishima elasticities in any existing AGE model and demonstrates a one-to-one numerical equivalence regardless of the elasticity measure employed. Replicating prior results, plausible parameter values are applied to the reformulated model to analyze the source-side, distributional effects of a pollution tax and highlight how the Allen and Morishima elasticities differ.

This paper develops a technique for constructing analytical general equilibrium (AGE) models using the Morishima elasticity of substitution (MES) to measure economic trade-offs in general production functions of three (or more) inputs. The traditional method for building such models employs the Allen elasticity of substitution (AES) (see, e.g., Mieszkowski, 1972; de Mooij and Bovenberg, 1998; Fullerton and Heutel, 2007; Fullerton and Monti, 2013). This new methodology is important because the MES and AES have different interpretations. Specifically, the AES determines if two inputs are price substitutes (or price complements), while the MES measures the change in relative inputs for a change in relative input prices and thus it is a "natural generalization of the Hicksian two-variable elasticity" (Blackorby and Russell, 1989, pp.885). This paper demonstrates that formulating an AGE model with Morishima elasticities provides additional insights beyond the traditional AES formulation.

Since the MES and AES have different interpretations two natural questions arise. First, can AGE models be formulated using Morishima elasticities (since such models have traditionally used Allen elasticities)? Second, if so, how does the interpretation of such models change when using the MES? To investigate these questions, this paper reformulates the model in Fullerton and Heutel (2007) that originally employs Allen elasticities to instead use Morishima elasticities. The MES formulation results are then compared and contrasted with the original AES formulation results. Key differences in favor of the MES formulation include: (1) fewer elasticity parameters making the closed-form solutions easier to interpret; (2) direct comparison of elasticity signs and magnitudes between production sectors with three (or more) inputs and two-input sectors due to the Hicksian interpretation; and (3) an intuitive special case equivalent to assuming a constant elasticity of substitution (CES) production function. In contrast, the AES formulation requires more parameters, makes it difficult to compare elasticity signs and values across sectors, and lacks a clearly interpretable set of assumptions leading to the useful special case of CES production.

The results in this paper are general in two different ways. First, they provide a template for constructing new AGE models using Morishima elasticities. Second, they allow a researcher to switch between MES and AES measures in any existing AGE model. That is, one can start with the closed-form solutions to any model using the traditional Allen elasticities and rewrite those closed-form solutions to instead use Morishima elasticities. By enabling the switch from an AES to MES formulation this paper helps connect two different literatures. The first literature estimates values of Morishima elasticities in many different contexts. For example, Koetse et al. (2008) provide a meta-analysis of over 100 Morishima capital-energy substitution elasticities. In another example, Considine and Larson (2006) estimate the Morishima elasticity between clean inputs (i.e. labor) and sulfur dioxide emissions at coal-fired power plants. The second literature employs Allen elasticities of substitution in AGE models (see citations above). Thus, once the elasticity switch is completed, estimated MES values can be used in models

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originally formulated with Allen elasticities to perform policy analysis. This is possible because there is a one-to-one numerical equivalence, between otherwise identical models, regardless of whether Allen or Morishima elasticities measure substitution in production.

This paper also demonstrates that the important special case of CES production follows directly from the MES formulation, but not the AES formulation. To start, Blackorby and Russell (1989) show that assuming symmetric Morishima elasticities is equivalent to assuming CES production, and their result is applied here to analytical general equilibrium models. Moreover, CES production is a standard assumption in the computational general equilibrium (CGE) literature (see, e.g., Shoven and Whalley, 1984; Bergman, 2005). Thus, the MES formulation connects AGE models to CGE models. Furthermore, this paper proves AGE models using the AES formulation can only yield CES production under restrictive assumptions about the relationship between the Allen elasticities and the input costs shares. However, CGE models are calibrated using observed data such as the input cost shares (Mansur and Whalley, 1984), and thus the connection between AGE models using Allen elasticities and CGE models employing CES production functions is limited.

The paper proceeds as follows. Section 1 provides background about AGE models and the elasticity measures examined in this paper (MES vs. AES). Importantly, this section also provides an explicit expression relating the Morishima and Allen elasticities. Section 2 constructs a reformulated version of the Fullerton and Heutel (2007) model using the MES. Section 3 examines the new, closed-form solutions to show how the MES formulation provides additional insights. Interpretation is aided by solving a simpler model that collapses the three-input production sector of the full model into a two-input sector (via a composite input). Section 4 explores the special case of CES production enabled by the MES formulation along with another special case that eliminates output effects (and these two cases alone resolve all the special cases in Fullerton and Heutel, 2007). Also, contrasting the special cases shows that potentially perverse results in model arise from substitution effects alone. Finally, this section provides a proof that the CES production under the AES formulation requires restrictive assumptions on the relationship between the elasticities and the input cost shares. Section 5 replicates the policy analysis from Fullerton and Heutel (2007) using their parameter values adapted to the MES formulation. The analysis focuses on the sources-side, distributional effects of a pollution tax. This section also confirms the one-to-one numerical equivalence regardless of elasticity measure and highlights differences between the Morishima and Allen elasticities. Section 6 briefly concludes.

1. Background

This section has two parts. The first part provides background about analytical general equilibrium models and their connection to computational general equilibrium models. The second part provides background regarding the Morishima and Allen elasticities of substitution, and provides an important identity linking the two elasticities.

1.1. Analytical general equilibrium models

Harberger (1962) introduces an important class of analytical general equilibrium (AGE) models. Jones (1965) is an early example of a similar type of model. Originally used to evaluate the incidence of a corporate income tax, the Harberger model has been used over the past 50 years to investigate many different issues ranging from the incidence of the local property tax (Mieszkowski, 1972) to the double-dividend hypothesis in environmental policy (Bovenberg and De Mooij, 1994). Meanwhile, other research extends Harberger's original model by relaxing initial assumptions (e.g. Rapanos, 1986). Then, following the innovation of Mieszkowski (1972), when production functions have three (or more) inputs, Harberger-style models employed the Allen elasticity of substitution to measure trade-offs in production when representing a general functional form. The main benefit of AGE models is that they can provide closed-form solutions that help identify key economic mechanisms, but they are limited in dimensionality – for instance, the number of production sectors – to remain tractable.

As a response to the limited dimensionality of AGE models and in conjunction with the advent of low-cost, high-speed computing, computational general equilibrium (CGE) models became an increasingly popular tool (e.g. Shoven, 1976). The main benefit of CGE models is that their dimensionality can increase far beyond that of AGE models and thus, in general, such models are able to better quantify economic variables (Bergman, 2005). However, CGE models cannot provide closed-form solutions and often require the assumptions about specific functional forms such as nested CES production functions. Since CES production is a special case of the general model with Morishima elasticities and the results here are related to CGE modeling.

1.2. The Morishima elasticity of substitution

The Morishima elasticity of substitution (MES) was independently discovered by Morishima (1967) and Blackorby and Russell (1975), as (Morishima, 1967) was published in Japanese only. The main insight of Morishima (1967) is that elasticities of substitution for input ratios are generally asymmetric given more than two inputs, where it becomes important which input price changes for all ordered, pairwise combinations of inputs. Importantly, Blackorby and Russell (1989) demonstrate that the MES is a natural generalization of the (Hicks, 1932) two-input elasticity of substitution in settings of three or more inputs, and subsequently gained notoriety (Anderson and Moroney, 1993). While Blackorby and Russell (1989) discuss the MES in a partial equilibrium setting, this paper extends the MES's application to general equilibrium models (see Section 2).

Following Stern's (2011) notation, the definition of the MES between inputs i and j (for $i \neq j$), and denoted m_{ij}, is given:

$$m_{ij} \equiv \frac{\partial \ln\left(C_i(y,p)/C_j(y,p)\right)}{\partial \ln\left(p_j/p_i\right)} = \frac{\partial \ln\left(x_i/x_j\right)}{\partial \ln\left(p_j/p_i\right)} \tag{1}$$

where y is the output quantity, p is the vector of input prices, and p_i is the price of input i. Also, the cost function is $C(y, p)$, so that C_i is the derivative of the cost function with respect to the i^{th} price. Recall Shepard's Lemma states that $C_i(y, p) = x_i$, where x_i is the input quantity demanded for input i conditional on output level y and prices p. Then, the MES is an elasticity that measures the change in an input ratio with respect to a change in the corresponding input price ratio, while holding output constant and also letting all other input quantities adjust optimally but holding their prices constant.

For the MES, importantly, the input price ratio only changes by varying one of the prices. That is, m_{ij} measures the change in the input ratio, x_i/x_j, resulting from an adjustment in the input price ratio, p_j/p_i, due to p_i changing (and thus a "two-factor–one-price" elasticity in the taxonomy of Mundlak (1968)). For instance, if p_i increases then the price ratio p_j/p_i falls, and if $m_{ij} > 0$, then the input ratio x_i/x_j falls too, meaning the producer substitutes away from the relatively more expensive input x_i toward the relatively less expensive input x_j. Mechanically, if the denominator in Eq. (1) is negative and $m_{ij} > 0$, then the numerator must be negative too. Thus, if $m_{ij} > 0$, then inputs i and j are Morishima substitutes with respect to p_i; and, if $m_{ij} < 0$, then the two inputs are Morishima complements. Although, for any pair of inputs i and j only one of the Morishima elasticities can be negative, so if $m_{ij} < 0$ then it must the case that $m_{ji} > 0$ (Stern, 2011).

While the terms "Morishima substitutes" and "Morishima complements" are not standard in the literature, other researchers use the terms "complements" and "complementarity" to describe pairs of inputs with a MES value less than zero. For instance, when describing their results Considine and Larson (2006) say, "Higher prices for high-sulfur fuel prices also significantly reduce the ratio of emissions to