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A wavelet analysis of US fiscal sustainability

Iolanda Lo Cascio *

Department of Economics, Business and Statistics, University of Palermo, Viale delle Scienze, 90128 Palermo, Italy

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ABSTRACT

In this paper, we reassess the relationship between primary deficit and lagged debt to GDP ratio (Bohn, 1998), to test for US debt sustainability over the period 1795–2012. Our analysis is rooted in the wavelet domain enabling the detection of interesting patterns and otherwise hidden information. We find evidence of long term fiscal sustainability but only up until 1995 and also we show that governments tend to respond more vigorously to budget deficits when the level of debt is high rather than low.

1. Introduction

In this paper, we reassess the relationship between primary deficit and lagged debt to GDP ratio (Bohn, 1998), to test for US debt sustainability over the period 1795–2012. Our analysis is rooted in the wavelet domain enabling the detection of interesting patterns and otherwise hidden information. The issue of US long run fiscal sustainability has been widely debated and controversial since the seminal work of Hamilton and Flavin (1986). These authors provided an empirical framework for testing the government's intertemporal budget constraint (IBC) in the form of a non-positive discounted value of the outstanding debt (*i.e.*, transversality condition, TC) and describe sustainability in terms of the ability for a government to pay back its debt with the discounted sum of the primary surpluses generated in the future.

Using U.S. data, Hamilton and Flavin (1986) found stationarity in the debt processes and concluded that the government's borrowing constraint was not violated.

Another common approach in literature is to investigate the presence of a cointegration relationship between government revenues and spending. Cointegration-based tests (Trehan and Walsh, 1988; Hakkio and Rush, 1991; Haug, 1991; Quintos, 1995; Ahmed and Rogers, 1995) provide a richer interpretation in terms of degree of sustainability. Trehan and Walsh (1988) show that government spending, inclusive of interest payments, and government revenues should be cointegrated with a cointegrating vector equal to $[-1, 1]'$. Hakkio and Rush (1991) relaxed this condition and showed that cointegration and $0 < \beta < 1$ are necessary conditions for (a strict form of) sustainability. Quintos (1995) argues that the condition on β to be less than 1 is a necessary and sufficient condition for deficit sustainability and that cointegration is only a sufficient one. However fiscal deficit (especially when expressed as GDP percentage) may result to be not sustainable (for $\beta < 1$) because in such a case expenditures grow faster than revenues. That situation corresponds to a weak form of sustainability because it is inconsistent with the government's ability to market its debt in the long run.

Bohn (1998) highlights the difficulty of standard unit root and cointegration tests of rejecting the consistency of the data with the IBC and, in a later paper (Bohn, 2007) proves that if a debt series is integrated of order m $D_t \sim I(m)$ for any finite m, then debt satisfies the transversality condition and debt, revenues, and spending satisfy the intertemporal budget constraint.

Berenguer-Rico and Carrion-i-Silvestre (2011) work in an I(2) stochastic process framework implied by the stock flow system of the fiscal variables and are able to distinguish among sustainability through deficit or through debt. Moreover they develop a test for regime shifts in I(2) cointegration and multicointegration relationships. They also configure sustainability through debt as a deeper concept of sustainability in which the government is equilibrating not only the flows but also the stocks in the long run.

In contrast to the literature analyzed so far and following the observation that "*Historically interest rates on "safe" U.S. government bonds have been significantly below the average rate of economic growth*", Bohn (1995) uses a stochastic model where a period t stochastic discount factor, $u_{t,n}$, is used for discounting state-contingent primary surpluses S in period t + n and it is determined by the distribution of revenues and spending across states of nature. It, thus, can be interpreted as investor's marginal rate of substitution between periods t and t + n. Given that both $u_{t,n}$ and the primary surplus S_{t+n} are stochastic, the expected value of their product contains a covariance term.

Therefore, the current stock of public debt, for which intertemporal solvency holds, takes into account the infinite sums of these two terms.[1]

* Tel.: +39 0 91 23895268.
E-mail address: iolanda.locascio@unipa.it.

[1] More formally, $D_t = \sum_{i=1}^{\infty} E_t(u_t, n) * E_t(S_t + n) + \sum_{n=1}^{\infty} cov_t(u_t, n, S_t + n$.

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As long as surpluses co-vary positively with $u_{t,n}$, hence with systematic risk included in the stochastic discount factor, the intertemporal solvency constraint can be consistent with primary surpluses that are negative on average. *Ad hoc* sustainability test ignores the covariance term in the equation above and relies only on exploring the unit-root and co-integration features of fiscal data. Bohn (1998) introduces a simple method to check if governments are taking corrective actions. According to him, a sufficient condition for debt sustainability is the existence of a fiscal reaction function in which the response coefficient of the primary surplus to public lagged debt stock, both to GDP ratio, after controlling for other determinants of the primary balance (*i.e.*, the cyclical position of GDP and government purchases), is statistically significantly positive. The intuition behind is that, following an increase in the size of the outstanding debt stock, policy-makers adopt changes to increase revenues and/or cut expenditures so as to increase the surplus in order to prevent the debt ratio from exploding. Unit root tests based on univariate regressions are thus misleading because, by not adjusting for war time spending and cyclical fluctuations, they make it difficult to reject the null and they obscure the positive response of primary surplus to the debt over GDP ratio.

The advantage of the model based approach is that it does not require any assumption on the interest rates[2] and it is valid for economies with uncertainty and risk aversion. Moreover it does not matter if government bond rates are below or above the growth rate.

We enter the debate on US debt sustainability by approaching it in a novel manner. Our analysis is not rooted in the time domain. Frequency domain analysis, sometimes, provides a more insightful representation of econometric data by decomposing it into sinusoidal components of various frequencies, which have intensities that vary across the frequency spectrum. We do not follow a pure frequency domain approach because neither mode of analysis is sufficiently flexible to cater for *truly evolving phenomena*, which may be subject to gradual drifts and to occasional abrupt changes.

To the best of our knowledge, apart from Cunado et al. (2004), who use a methodology based on fractional processes, no empirical work has been done in the frequency domain to test for US fiscal sustainability.

We are interested in detecting and quantifying the time–frequency dependence between primary balance and lagged debt to GDP ratios. For that purpose we make use of some wavelet analysis tools by analyzing, firstly, the univariate features of the series through the Continuous Wavelet Transform (CWT) (Grossman and Morlet, 1984; Mallat, 1998) and the wavelet power spectrum which give information simultaneously on time and frequency features of the data. Wavelet analysis reveals the spectral characteristics of a time series (discovering patterns and otherwise hidden information), in particular, the way in which different periodic components of the data on US national debt and primary balance evolve over time.

There are several advantages in the use of wavelet analysis compared to the traditional time-domain approaches.

On one hand, traditional unit root tests are biased toward the rejection of the null hypothesis when fiscal variables present a non-linear behavior (Sarno, 2001; Cipollini et al., 2009). Also Bohn (1998) documented that governments react more vigorously when fiscal imbalances are large rather than small. Another source of potential bias is the presence of structural breaks (Ahmed and Rogers, 1995; Quintos, 1995; Chortareas et al., 2008 among others). Cointegration based tests, aiming at testing the violation of the intertemporal solvency constraint, ignore the covariance term in the equation for debt. They focus on the relationship between government spending and revenue only at zero frequency and explore, within the time domain, if there is any evidence of (exogenous or endogenous) regime shifts.

On the other hand, the fiscal reaction function introduced and estimated for the US by Bohn (1998) takes into account the covariance term in the debt equation. The author investigates if there is a positive relationship between primary surplus and debt at horizons exceeding those characterizing the business cycle by using control variables as proxies of the cyclical component of real economic activity and/or government spending. Such proxies are previously estimated and, we argue, that they might be responsible of introducing a measurement error problem biasing the estimation results of the fiscal reaction function. Another critique moved to the *ad hoc* sustainability tests is that they rely on historical data while sustainability is a forward-looking concept. The two-sided nature of wavelet filters is particularly helpful in this case. Wavelet coherence tools (Grinsted et al., 2004) give a measure of the time-varying relation between the fiscal variables at several frequencies, especially the ones corresponding to and exceeding business cycle horizons but different from zero. In such a way we do not really need to augment our analysis with the variables motivated by Barro (1979) in the tax-smoothing model. Moreover we can also give insight on the possible presence of non-linearities by looking at the color contour plot of the coherence. In fact, stronger coherence might result, meaning that governments tend to respond more vigorously to budget deficits, when the level of debt is high rather than low.

The rest of the paper is organized as follows. The next section presents the methods of wavelet analysis which are, then, used in Section 3 to investigate the relationship between the fiscal covariates. Section 4 concludes.

2. Methodology

The popularity of wavelets with compact supports, such as those belonging to the Daubechies' family (1988, 1992), is due mainly to their relation to the dyadic multiresolution analysis that dominates wavelet research. Historically the continuous wavelet transform came first.

The continuous wavelet transform is defined as

$$W_x = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{s}} \psi^* \left(\frac{t-u}{s} \right) dt \tag{1}$$

where $\psi(u, s)$ is a mother wavelet from which a family of wavelet daughters is obtained by scaling and translating it, thus, by changing u (controlling the location) and s (determining the width of the wavelet and the frequency resolution); the star in Eq. (1) denotes the complex conjugate. It is an energy preserving transformation of the data which allows for perfect reconstruction and produces a variance decomposition with a good time localization of the time series under investigation.

Whenever the aim of the analysis is the study of cycles and periodicity it is useful to choose a complex-valued wavelet which gives rise to a complex-valued transform which can be separated into a real part and an imaginary one, $W_x(u,s) = |W_x(u,s)| e^{i\phi_x(u,s)}$ and contains information about amplitude and phase (*i.e.*, $\phi_{u,s}$). For that purpose it is useful to choose an analytic wavelet.[3] The most widely used among the analytic wavelets is the *Morlet* wavelet (Goupillaud et al., 1984). It is the one with the best trade-off between time and frequency precision and is defined as

$$\psi_{\omega_0}(t) = \frac{1}{\pi^1/4} e^{i\omega_0 t} e^{-t^{s2}/2}. \tag{2}$$

Here ω_0 is the central frequency of the wavelet. It has to be higher or equal than 5 in order to ensure that ψ is analytic. For $\omega = 6$ the frequency center is $\mu_f = 6/2\pi \approx 1$ (Aguiar-Conraria et al., 2008). As a consequence, due to the frequency scale relationship

$$f = \frac{\mu_f}{s} \approx \frac{1}{s} \tag{3}$$

[2] Usually the simplifying assumptions are $r_t = r > 0$ or $E_t[r_t + 1] = r > 0$.

[3] An analytic wavelet is a wavelet such that its Fourier transform is zero for negative frequencies $\omega < 0$.