Economic Modelling 51 (2015) 460–472

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

Modeling longevity risk transfers as Nash bargaining problems: Methodology and insights

Rui Zhou [a], Johnny Siu-Hang Li [b,*], Ken Seng Tan [b]

[a] Warren Centre for Actuarial Studies and Research, University of Manitoba, Winnipeg, MB, Canada
[b] Department of Statistics and Actuarial Science, University of Waterloo, Waterloo, ON, Canada

ARTICLE INFO

Article history:
Accepted 18 August 2015
Available online xxxx

Keywords:
Longevity risk
Securitization
Pricing
Pareto optimality
Nash's bargaining solution

ABSTRACT

The problem of longevity risk has recently received considerable attention. In this paper, we apply economic modeling methods to longevity risk securitization, which is now regarded by pension and insurance industries as a solution to the problem. Specifically, we model the trade of a longevity security as a two-player bargaining game, and use Nash's bargaining solution to determine the outcome of it. Our work not only offers an alternative method for pricing longevity securities, but also reveals several properties about the market for longevity securities. First, a trade would occur if the longevity security is an effective hedging instrument, and the trade would benefit all agents involved. Second, a trade of longevity risk can reduce pension plans' bankruptcy risk, safeguarding the financial security of pension plan members. Finally, compared to the competitive equilibrium, Nash's bargaining solution yields higher trading prices. Therefore, as the market becomes more competitive, pension plans may hedge longevity risk at a lower cost.

1. Introduction

The population of the developed world is living longer. In the study by Oeppen and Vaupel (2002), it is shown empirically that record female life expectancy since 1840 has been increasing at a steady pace of 2.43 years per decade, and that the trend has no sign of slowing down. The fact that everyone is living longer is a good news and success story, but unanticipated improvements in life expectancies can pose problems to individuals, corporations and governments. This risk, which is commonly referred to as longevity risk, has received considerable attention among applied economists in recent years (see, e.g., Bielecki et al., 2015a, 2015b; Koka and Kosempel, 2014; Kudrna et al., 2015).

More specifically, if mortality improves faster than expected, then governments and corporations offering defined-benefit pension plans to their employees will need to pay out more social security and pension benefits. Since about ten years ago, pension sponsors and annuity providers have started to consider securitization as a solution to the problem of longevity risk. This has led to the emergence of the 'Life Market', the traded market in assets and liabilities that are linked to human mortality. By trading in the Life Market, longevity risk can be transferred to parties who are interested in taking on the risk to earn a premium and to diversify their investment portfolios.[1]

The first longevity security was announced by BNP Paribas and the European Investment Bank in 2004. Since then, the Life Market has witnessed several longevity derivative transactions, most notably the £3 billion longevity swap that Rolls Royce transacted with Deutsche Bank in 2011 to cover the longevity risk of its pension plan. However, as exemplified by the following three characteristics, the Life Market is still in its early stage of development.

1. Relative to liquid trading financial markets, the number of participants in the Life Market is small.[2]
2. The products are inhomogeneous. Most transactions that took place in the Life Market were bespoke deals, customized to the hedgers' own risk characteristics.
3. We do not have perfect knowledge of the Life Market. The pricing information is often proprietary, and some transactions in the market are not even made known to the public.

When it comes to trading in the Life Market, both the seller and buyer need to estimate the price of security being traded. There exist a

* Corresponding author. Tel.: +1 (519) 888 4567x35542; fax: +1 (519) 746 1875.
E-mail addresses: rui.zhou@umanitoba.ca (R. Zhou), shli@uwaterloo.ca (J.S.-H. Li), kstan@uwaterloo.ca (K.S. Tan).

[1] Securities in the Life Market have very low correlations to other asset classes, such as foreign exchange, commodities, equities and credit. The low correlations between longevity and other risks have been discussed by Blake et al. (2006).

[2] According to Blake et al. (2013), there were only seven providers among the known longevity swap transactions in the U.K. from 2007 to 2012. The seven providers include Credit Suisse, Deutsche Bank, Goldman Sachs, J.P. Morgan, Legal & General, RBS and Swiss Re.

http://dx.doi.org/10.1016/j.econmod.2015.08.019

number of methods for pricing longevity securities, but these methods, as we now explain, may not be appropriate in the current stage of the Life Market.

Most of the prevailing pricing methods are based on the principle of no-arbitrage. Generally speaking, to implement these methods, we first estimate a distribution of future mortality rates under the real-world probability measure. Second, we identify a risk-neutral probability measure, under which existing securities in the market are correctly priced, and transform the real-world distribution to its risk-neutral counterpart. Finally, we can estimate the price of a newly introduced longevity security by discounting, at the risk-free interest rate, its expected payoff that is calculated using the risk-neutral distribution of future mortality rates.

The second step in the procedure above requires market price data as input. For instance, the pricing methods of Lin and Cox (2005), Denuit et al. (2007), Dowd et al. (2006) and Milidonis et al. (2011) make use of a distortion operator such as the Wang transform (Wang, 2000) to create a risk-neutral probability measure. In using these methods, there is a need to estimate the parameters in the distortion operator with prices of securities that are actually traded in the Life Market. Hence, in today's Life Market where market price data are very limited, it is not easy to implement these pricing methods. Other no-arbitrage pricing methods, including the constrained maximization of entropy (Li, 2010; Li and Ng, 2011; Li et al., 2011) and the use of a risk-adjusted two-factor mortality model (Cairns et al., 2006; Deng et al., 2012), are subject to the same problem.

Recently, researchers have considered economic methods for pricing longevity securities. In particular, Zhou et al. (2011, 2015) treat pricing in the Life Market as a Walrasian tâtonnement process, in which the price of a longevity security is determined through a gradual calibration of supply and demand. Other than the trading price, one can also estimate the trading quantity by using the demand and supply curves resulting from the tâtonnement process. In general, economic pricing methods do not entail the identification of a risk-neutral probability measure. This means that the use of these methods does not require prices of traded securities as input, thereby sparing us from the problems associated with the lack of market price data.

Nevertheless, a significant problem still remains. All previously mentioned pricing methods require the assumption of a perfectly competitive market. In a competitive market, there are a large number of participants, the products sold are identical, and each participant knows the price and quantity of the goods sold by everyone. The aforementioned characteristics of the current Life Market indicate that it is not even close to competitive. In the present stage of market development, prices calculated from the tâtonnement approach could be used as a benchmark in situations when no-arbitrage methods are difficult to implement, but they may not be regarded as accurate estimates.

In this paper, we relax the assumption of a perfectly competitive market by treating the trade of a longevity security as a two-player bargaining game. The competitive market assumption is waived, because rather than being a price taker, each player in the game can influence the price of the security being traded through the bargaining process. In our set-up, it is assumed that one party is a pension plan sponsor or an annuity writer who intends to hedge her longevity risk exposure, and that the other party is an investor who intends to take on the risk for a risk premium. The bargaining outcome is obtained through Nash's bargaining solution, an axiomatic solution proposed by Nash (1950) to solve the two-person bargaining game.

Nash's bargaining solution suppresses many details of the decision making process. It explains outcomes by identifying conditions that any outcome arrived at by rational decision makers should satisfy a priori. These conditions are treated as axioms, from which the outcome is deduced by using set-theoretical arguments. It is therefore not computationally difficult to find the outcome of the game, that is, the trading price and quantity, by using Nash's bargaining solution. Besides being easy to implement, the pricing framework proposed in this paper also preserves many advantages of the tâtonnement pricing approach, including the provision of a unique trading price and quantity.

Nash's bargaining solution has been considered extensively by applied economists (see, e.g., Kamijo and Tomaru, 2014; Nakamura and Takami, 2015). It has also been previously applied to problems in insurance. Borch (1974) applied Nash's bargaining solution to reciprocal reinsurance treaties, and used it to determine the quota ceded by each player. Kihlstrom and Roth (1982) and Schlesinger (1984) studied how insurance contracts are reached through Nash bargaining, and investigated the effect of risk aversion on the outcome of bargaining about the terms of an insurance contract. The work of Kihlstrom and Roth (1982) was subsequently revisited by other researchers including Viaene et al. (2002) who considered alternative set-ups. Boonen et al. (2012) defined a cooperative non-transferable utility game for the optimal redistribution of longevity risk between pension funds and life insurers.[3] To the best of our knowledge, this paper is the first to model the trade of a longevity security as a two-person bargaining game and to solve it with Nash's bargaining solution.

In addition to theoretically presenting how a bargaining game can be applied to modeling trades in the Life Market, we strive to provide practical implications through a numerical illustration of a trade between a pension plan sponsor and an investor. In particular, we examine how such a trade under the bargaining game framework may benefit the participating agents, and identify the factors that largely determine the trading results. We also compare the outcomes from Nash's bargaining solution with those from the competitive market equilibrium and the Kalai–Smorodinsky bargaining solution, with an aim to provide market participants and applied economists information about what may possibly happen when the market condition or game setting changes.

The remainder of this paper is organized as follows. Section 2 presents the set-up of the trade of a longevity security between a hedger and an investor. Section 3 details how the trade can be modeled as a two-person bargaining game, and explains how Nash's bargaining solution can be used to determine the outcome of the trade. Section 4 studies how the modeling of the trade would be different if the Life Market is perfectly competitive. Section 5 applies the theoretical results from the previous two sections to a hypothetical trade. Section 6 investigates the conditions of Pareto optimality, and compares the outcomes arising from Nash's bargaining solution and the competitive equilibrium. Finally, Section 7 concludes the paper.

2. Setting up the trade

In this paper, we model the trade of a mortality-linked security between two economic agents, namely Agents A and B.

Agent A could possibly be an annuity provider or a pension sponsor, who has an exposure to longevity risk. Suppose that Agent A has annuity or pension liabilities that are due at times 1, 2, …, T. The amount due at time t is $f_t(Q_t^L)$, which is a deterministic function of Q_t^L, where Q_t^L is an index that contains information about the mortality of the population associated with Agent A's annuity or pension liability up to and including time t. At time 0, the values of Q_t^L for $t > 0$ are not known and are governed by an underlying stochastic process.

The risk that Agent A is facing is a variable risk, as it can result in either profit or loss. In particular, she suffers a loss if mortality improves faster than expected (as more payments have to be made), but makes a profit if the opposite is true. The profit arising from a favorable deviation from the expected mortality experience is referred to as mortality profit

[3] Boonen et al. (2012) found that under certain assumptions, proportional risk redistribution is optimal. Proportional risk redistribution may be achieved by means of reinsurance, but not through the trade of standardized securities such as longevity bonds and mortality forwards in the Life Market.