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Modeling high-frequency volatility with three-state FIGARCH models

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ABSTRACT

Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity (FIGARCH) models have enjoyed considerable popularity over the past decade because of their ability to capture the features of volatility clustering and long-memory persistence. However, in the presence of structural changes, it is well known that the estimate of long memory will be spurious. Consequently, two modeling approaches are developed to incorporate structural changes into the FIGARCH framework. One approach is to model the intercept in the conditional variance equation via a certain function of time. Based on this approach, the Adaptive-FIGARCH (A-FIGARCH) and Time-Varying FIGARCH (TV-FIGARCH) models are proposed. The second approach is to model the time-series in separate stages. In the first stage, a certain algorithm is applied to detect the change points. The FIGARCH model is fitted to the time-series in the next stage, with the intercept (and other parameters) being allowed to vary between change points. An example of a recently developed algorithm for detecting change points is the Nonparametric Change Point Model (NPCPM), which can be readily applied to the standard FIGARCH framework (NPCPM-FIGARCH). In this paper, we adopt the second approach but use the Markov Regime-Switching (MRS) model to detect the change points and identify three economic states depending on the scale of volatility. This new 2-stage Three-State FIGARCH (3S-FIGARCH) framework is compared with other FIGARCH-type models via Monte-Carlo simulations and high-frequency datasets. From the comparison, we find that the 3S-FIGARCH model can largely improve the fit and potentially lead to a more reliable estimator of the long-memory parameter.

1. Introduction

Persistence of a time series describes how fast the effect of current shock will die away. It has been extensively observed and studied in various fields of economics and finance in the past few decades (Aggarwal et al., 1999; Fan et al., 2008; Granger and Hyung, 2004; Jensen, 2000; Narayan and Narayan, 2007; Narayan and Narayan, 2011). The analysis of persistence can help researchers understand how the time series evolves and improve the forecasting quality (Franses and van Dijk, 1996; Ho et al., 2013; Liu, 2000; Narayan and Sharma, 2014; Westerlund and Narayan, 2012). In particular, the long-memory persistence describes the property that the effects of shocks last far longer than the usual autoregressive moving average (ARMA) process (Baillie and Morana, 2009; Baillie et al., 1996; Belkhouja and Boutahary, 2011; Bollerslev and Mikkelsen, 1996; Diebold and Inoue, 2001). A widely accepted definition of long memory is $var(S_T) = O(T^{2d+1})$, where $S_T = \sum_{t=1}^{T} y_t$, $\{y_t\}$ is a sequence of financial series and T is the number of observations (Diebold and Inoue, 2001). Then d is the long-memory parameter, and a positive value suggests the existence of long memory. Among recent finance studies, there is growing evidence suggesting that the long-memory persistence significantly exists in the volatility of financial return series (Barunk and Dvoráková, 2015; Caporale and Gil-Alana, 2013; Li, 2012).

In particular, the time-varying volatility of financial returns has been a considerable field of research since the introduction of the GARCH model. To incorporate long-memory persistence within this framework, the Fractional Integrated GARCH (FIGARCH) model is then proposed (Baillie et al., 1996; Bollerslev and Mikkelsen, 1996). The FIGARCH model has thus received considerable interest because of its ability to capture the long-memory persistence in the volatility (Baillie and Morana, 2009; Belkhouja and Boutahary, 2011; Ho et al., 2013). Despite this ability, it has the same main weakness of the original GARCH model, which is the assumption that the conditional volatility has only one regime over the entire period. However, many studies demonstrate that structural changes are common in financial datasets (Beltratti and Morana, 2006; Engle and Rangel, 2008). Further, Diebold and Inoue (2001) argue that the existence of structural changes or stochastic regime switching is not only related to long memory but also easily confused with it. This finding is supported by many empirical studies, where spurious long-memory is found when structural changes are present (Granger and Hyung, 2004; Mikosch and Starica, 2004; Yalama and Celik, 2013). As a result, many researchers have suggested

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that structural changes should be incorporated into long-memory models to properly fit financial return volatility (Baillie and Morana, 2009; Beine et al., 2001; Belkhouja and Boutahary, 2011; Martens et al., 2004; Morana and Beltratti, 2004). For instance, the Adaptive FIGARCH (A-FIGARCH) model developed by Baillie and Morana (2009) and the Time-Varying FIGARCH (TV-FIGARCH) model developed by Belkhouja and Boutahary (2011) allow the intercept in the conditional variance equation to be time-varying. Essentially, this is achieved by modeling the intercept via parametric functions.

Among the existing literature, another approach to incorporate the structural changes in the GARCH-type framework is to fit the model in stages (Aggarwal et al., 1999; Malik et al., 2005; Ross, 2013). First, the return series is fitted by a certain algorithm to detect the abrupt change points. The intercept (and other parameters) in the conditional variance equation is then allowed to be different for each period between the change points. In the first stage, the most widely employed method is the Iterated Cumulative Sum of Squares (ICSS) algorithm proposed by Inclan and Tiao (1994). However, as pointed out by Ross (2013), the original ICSS only works for Gaussian distribution. To overcome this problem, Ross (2013) develops the Nonparametric Change Point Model (NPCPM) algorithm, which employs the Mood test Mood (1954) to detect the change points. The NPCPM-GARCH model is then proposed, which can effectively work for both Gaussian and non-Gaussian data. This approach can be straightforwardly extended to the FIGARCH framework. A potential problem of the NPCPM algorithm is that it identifies the change points without considering the economic states. For example, sample periods with different structures are detected based on the change points only, but they will not be combined and studied subsequently according to their economic similarity. This may lead to a model that lacks parsimony because economic similarity is neglected. Besides, instead of assuming that the volatility series will switch back and forth between different regimes with some probability, the NPCPM algorithm assumes that the switch to a different regime is permanent. In addition, as NPCPM requires the return series to be independent, Ross (2013) suggests that it should be applied to the standardized residuals from the (FI)GARCH model. This might cause some problems such as the lack of economic interpretation of the detected change points and loss of information.

In this paper, we propose a two-stage Three-State FIGARCH (3S-FIGARCH) model, which also incorporates the structural changes by modeling the FIGARCH process in stages. In the first stage, the MRS framework proposed by Hamilton (1989) is employed to detect change points directly. The MRS model assumes that there are two economic states (low- and high-volatility states) in the financial return series. Also, the series can switch between the states over time, and the state process is a stationary, irreducible Markov process. We further use the three-state classification proposed by Wilfling and Maennig (2001) and Wilfling (2009) to classify the underlying state process: calm (extremely low volatility), turbulent (extremely high volatility) and intermediate (others). In the second stage, parameters of the FIGARCH process are allowed to be different for each state. As there are only three possible values for each parameter, our model should be more parsimonious than the NPCPM-FIGARCH framework. Moreover, the MRS model does not require the financial return series to be originally independent, so that the detected states (change points) are more reliable. Finally, the MRS model takes the economic information (low and high volatility) of the return series into consideration, and the detected states can have meaningful economic interpretation. To demonstrate the usefulness of the model, we firstly conduct a series of simulation studies. It is shown that the 3S-FIGARCH model outperforms the other structure-changing specifications (A-, TV- and NPCPM-FIGARCH) in all cases.

We also compare their performance via empirical studies on four world stock indexes. They are hourly data collected from 1 January 2001 to 31 December 2012, including: (1) the NASDAQ, which consists of over 3000 stocks listed on the NASDAQ stock market; (2) the DAX, which consists of 30 large Germany companies; (3) the Nikkei, which consists of 225 Japanese companies; and (4) the ASX, which consists of 50 large Australian companies. Assumptions of Gaussian, Student's t and General Error (GED) distributions are modeled individually for each model and stock index.[1] The results suggest that models with non-Gaussian distribution assumptions outperform models with Gaussian distribution assumptions. More importantly, the 3S-FIGARCH specification generally gives a better fit to the data when measured using standard model selection criteria. It also provides a potentially more reliable estimate of the long-memory parameter. Thus, our 3S-FIGARCH framework could be a widely useful tool for modeling the long-memory persistence of high-frequency financial volatility in other contexts.

The remainder of this paper proceeds as follows. Section 2 describes the existing and structure-changing FIGARCH models, including FIGARCH, A-FIGARCH, TV-FIGARCH and NPCPM-FIGARCH, as well as the likelihood functions for Gaussian, Student's t and GED distribution assumptions. Section 3 explains the 3S-FIGARCH model proposed in this paper and compares its performance with other FIGARCH-type models via a series of simulation studies. We discuss the empirical results in Section 4. Section 5 concludes the paper.

2. The original and existing structure-changing FIGARCH models

2.1. The original FIGARCH model

The FIGARCH model proposed by Baillie et al. (1996) is extended from the family of GARCH models. In addition to the features of incorporating volatility clustering and providing good in-sample estimates (Franses and van Dijk, 1996; French et al., 1987), FIGARCH is particularly designed to model the long-memory persistence of financial volatility.

The original FIGARCH$(1, d, 1)$ model is specified as follows:

$$\begin{aligned} &r_t = \mu + \varepsilon_t \text{ and } \varepsilon_t = \eta_t \sqrt{h_t} \\ &b(L)h_t = \omega + \left[b(L) - \phi(L)(1-L)^d\right]\varepsilon_t^2 \\ &b(L) = 1 - b_1 L \text{ and } \phi(L) = 1 - \phi_1 L \end{aligned} \tag{1}$$

where ε_t is the error at time t, h_t is the conditional volatility of ε_t at time t, η_t is an identical and independent sequence following a specific distribution, L is the lag operator, $(1-L)^d$ is the fractional differencing operator as defined by Hosking (1981) and d is the long-memory parameter. We have a stationary long-memory process for volatility when $0 < d < 1$. If $d = 1$, the process has a unit root and thus a permanent shock effect, which is equivalent to the IGARCH model. If $d = 0$, the process reduces to an ordinary GARCH process without long-memory persistence (Baillie et al., 1996).

2.2. A-FIGARCH model

To control for the effects of structural changes, Baillie and Morana (2009) suggest that the intercept of the conditional variance equation should be time dependent. Based on Andersen and Bollerslev's (1997) flexible functional form, Baillie and Morana (2009) propose the A-FIGARCH model, and its conditional variance equation is shown below:

$$\begin{aligned} &b(L)h_t = \omega + \left[b(L) - \phi(L)(1-L)^d\right]\varepsilon_t^2 + \omega_t \\ &\text{and } \omega_t = \sum_{j=1}^{k}\left[\gamma_j \sin(2\pi jt/T) + \delta_j \cos(2\pi jt/T)\right] \end{aligned} \tag{2}$$

[1] Although the FIGARCH model is originally based on Gaussian distribution (Baillie et al., 1996), significant evidence suggests that the financial return series is rarely Gaussian but typically leptokurtic and exhibits heavy-tail behavior (Bollerslev, 1987; Ho et al., 2013; Lin and Fei, 2013; Susmel and Engle, 1994). Student's t-distribution and GED are two widely used alternatives in finance study (Chkili et al., 2012; Fan et al., 2008; Ho et al., 2013; Zhu and Galbraith, 2011) and are employed in this paper to be compared with the Gaussian distribution.