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Fiscal policy tracking design in the time–frequency domain using wavelet analysis

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In this paper discrete wavelet filtering techniques are applied to decompose macroeconomic data so that they can be simultaneously analyzed in both the time and frequency domains. The *MODWT* (Maximal Overlap Discrete Wavelet Transform) is applied to US quarterly GDP data to obtain the underlying cyclical structure of the GDP components. A MATLAB program is then used to design optimal fiscal policy within an *LQ* tracking model with wavelet decomposition, and the results are compared with an aggregate model with no frequency decomposition. The results show that fiscal policy is more active under the wavelet-based model, and that the consumption and investment trajectories under the aggregate model are misaligned. We also simulate *FHEC* (Frequency Harmonizing Emphasis Control) strategies that allow policymakers to concentrate the policy thrust on tracking frequencies that are optimally aligned with policy goals under different targeting priorities. These strategies are only available by using time–frequency analysis. This research is the first to construct fiscal policy in an applied optimal control model based on the short and long cyclical lag structures obtained from wavelet analysis. Our wavelet-based optimal control procedure allows the policymaker to construct a pragmatic tracking policy, avoid suboptimal policies gleaned from an aggregate model, and reduce the potential for destabilization that might otherwise result due to improper thrust and timing.

1. Introduction

The macroeconomic accelerator model has proved to be a useful theoretical and empirical tool since Samuelson (1939) and Chow (1967). Kendrick (1981), Kendrick and Amman (2010), Kendrick and Shoukry (2013), and Hudgins and Na (2014) have all modeled quarterly fiscal policy within an applied macroeconomic optimal control *LQ* (Linear–Quadratic) tracking framework. Kendrick (1981) compared the deterministic model results to the closed-economy performance of the US under stochastic and adaptive control. Kendrick and Amman (2010), Kendrick and Shoukry (2013), and Hudgins and Na (2014) all present strong cases for the improvements of implementing a quarterly fiscal policy rather than an annual policy. Kendrick and Shoukry (2013) simulate the tracking performance and debt structure of the quarterly and annual models within a closed-economy macroeconomic model that contains monetary and interest rate components. Hudgins and Na (2014) examine optimal robust policies using a similar model for the US economy, but without money and interest rates in the model.

In each of these models, the particular lag structures in the policy variable design and in the consumption and investment functions could have destabilizing effects on the response dynamics. All of these analyses note that it is important for modelers to consider different lag structures in these equations in order to simulate the optimal policies under various parameters. Instead of estimating various time series models with alternating lags, another approach is to utilize wavelet analysis in order to gain insight into the lag structure through the time–frequency domain. Crowley and Hughes Hallett (2014) use an *MODWT* (Maximal Overlap Discrete Wavelet Transform) to find the frequency domain cyclical decomposition of US GDP component data for the period 1947–2012. This analysis will build the control model using the results obtained from a similar wavelet estimation strategy used by Crowley and Hughes Hallett (2014).

This will also set the stage for the optimal timing of the quarterly fiscal policy impulses. Kendrick and Shoukry (2013) select the first quarter as the period when government appropriations are determined for the year, and then compare this annualized scenario to a counterfactual situation where appropriations can be made during each quarter. They note that selection of other quarters for annual fiscal policy change might result in somewhat different performance when comparing quarterly to annual fiscal policy scenarios. However, the cyclical timing and lag structure can be determined with greater precision by using the quarterly lag structure gathered through wavelet filters in the time–frequency domain.

Using the longer cycles obtained from wavelet analysis also addresses the findings of recent neoclassical research. This is consistent

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with Leeper et al. (2010), for example, who find that the speed of the fiscal adjustment impacts the policy effectiveness. In addition, Leeper et al. (2010) find that government investment with comparatively weak productivity can dictate a contractionary government investment policy in the long-run. Crowley and Hughes Hallett (2014) use an *MODWT* wavelet decomposition and find that US fiscal policy has not been destabilizing or procyclical over the various business cycle frequencies; however, they also found that it has not been effective as a countercyclical stabilizer either. Designing quarterly fiscal policy rules that are built upon the full range of short and long-term cyclical wavelet components avoids the bias that might otherwise be introduced through inadequate recognition of the interplay of the short-term lags with the long-term cyclical components.

1.1. Purpose and scope

The purpose of this paper is to construct optimal fiscal policy within an optimal *LQ* (Linear–Quadratic) tracking control model that is formulated within the time–frequency domain based on the *MODWT* wavelet decomposition. This is the first research to integrate optimal control and discrete wavelet analysis in order to design macroeconomic policy. Section 2 examines the *MODWT* wavelet methods and the decomposition of data in the time–frequency domain, using data from the period 1947–2015. Section 3 builds and estimates a macroeconomic time–frequency accelerator model that is used within an optimal control system to determine optimal control feedback rules for fiscal policy. We convert the *LQ* tracking design into an *LQ* regulator design using the method employed by Hudgins and Na (2014), and develop a MATLAB software program to compute the optimal fiscal policy. This framework allows the policymaker to render deterministic, stochastic *LQG* (Linear–Quadratic Gaussian) and robust controller designs, but the research presented in this paper only presents simulations for the deterministic *LQ* tracking control design.

Section 4 uses the time–frequency model developed in Section 3 to run simulations that compare the optimal fiscal policy trajectories under the wavelet decomposition model with fiscal policy under the aggregate model with no decomposition. These simulations show that fiscal policy will be more active within the wavelet decomposition framework than in the model without decomposition. The simulations also show that the aggregate model would consistently produce consumption levels that are overvalued relative to consumption levels in the wavelet-based model, and would produce investment levels that are undervalued relative to the wavelet-based model throughout the planning horizon. This demonstrates the suboptimal fiscal policy and the errors in the projected consumption and investment trajectories that would likely result from using an aggregate model with no decomposition.

The government debt resulting from using the optimal wavelet policy is similar to the debt levels under the aggregate model, but the comparative levels depend on the relative priorities of the targeted state and control variables. When the tracking errors for government spending are assigned a high relative weight, the debt stock in the wavelet model at the end of the planning period will be slightly lower than debt that results from the aggregated model. When consumption and investment are more heavily weighted, the final debt stock is slightly higher under the wavelet model. Thus, another advantage of employing the wavelet-based system is the increased ability to project the impact of fiscal policy on the government stock of debt.

The simulations also explore *FHEC* (Frequency Harmonizing Emphasis Control) strategies. This approach allows the fiscal policymakers to place different weights on the tracking errors for government spending, consumption, and investment, at different frequency ranges. This is not possible under an aggregate model without time–frequency decomposition. For example, the optimal *FHEC* policy allows the government to place more emphasis on the time horizon where spending is targeted, and/or to place more emphasis on the consumption and investment intervals that will be most affected. This allows for an entirely new operational procedure for packaging the optimal fiscal policy, and for determining the likely effects at each frequency range. We simulate the model under three different *FHEC* policymaker priorities: (1) long-term business cycle and productivity targeting, (2) political cycle targeting, and (3) short-term stabilization targeting. The results show that fiscal policy can be structured to effectively reduce consumption and investment gaps for the cycles over the frequency ranges that are most emphasized under the different objectives.

Our analysis shows that the optimal policy under the wavelet model is more active in the sense that the trajectory has much greater variation than the policy under the aggregated model. When compared to a robust design, the finding that optimal policy is overly passive, or rigid, under a standard deterministic or stochastic control model is consistent with some of the findings in the robust control literature (Bernhard, 2002; Dennis et al., 2009; Diebold, 2005; Hudgins and Na, 2014; Onatski and Stock 2002). The more variable wavelet-derived policy shows that these robust-derived policies may have a stabilizing effect when the system is sub-optimally modeled due to the failure to employ time–frequency decomposition through wavelet filtering. The wavelet decompositions employed here increase the policy activism, but reduce the uncertainty in modeling procedure by providing a more complete model for the policymaker. The wavelet model thus reduces the need to rely on robust considerations in the control formulation. The wavelet-based model has extended Hudgins and Na (2014), so that it can be used in conjunction with robust control methods.

2. *MODWT* wavelet analysis

Here we provide a brief overview of the mathematical background of time–frequency analysis and the wavelet methodology. As mentioned previously, time domain analysis cannot provide a proper basis for analysis when frequencies are changing; in other words, time domain methods generally cannot reveal valuable and helpful information which are hidden in different frequencies. The most well-known frequency domain method is the Fourier Transform (*FT*). This method transforms time series data from the time domain to the frequency domain, but the problem with the *FT* approach is that the data is transformed into just the frequency domain, so there is no ability to simultaneously analyze relationships in both the time and frequency domains. Indeed, with the *FT* method, the time series under consideration should be locally and globally stationary; but unfortunately, this also imposes a limitation since a significant number of economic and financial time series are locally and globally nonstationary. This is because of trade-offs between the frequency resolution and time resolution (Weedon, 2003) and the *FT* cannot capture this properly. Although the *Windowed FT* is often used to address these shortcomings with the *FT*, it does so only partially, as it suffers from a further problem which lies in the Heisenberg uncertainty principle.[2]

2.1. Wavelet analysis

Wavelet analysis has its roots in multiscale decomposition, the so-called multiscale analysis or multiresolutional analysis, which was developed by Meyer (1986), Mallat (1989a,b), Strang (1989), and Daubechies (1992). Technically speaking, multiscale analysis is an approximation operation through a dense vector space (Hilbert space) with empty intersects from coarsest to less detailed information.

Following Mallat's explanation for the pyramid algorithm and multiresolutional analysis, the value of a variable x at time instant k, x_k, can be written as follows:

$$x_k \approx S_{J,k} + d_{J,k} + d_{J-1,k} + \ldots + d_{1,k} \tag{1}$$

where $d_{j,k}$ are detail components (wavelet "crystals"), $j = 1,\ldots,J$; $S_{J,k}$ is a trend component (the wavelet "smooth"); and J stands for the number

[2] For more information on orthogonal transforms, see Wang (2012) pp. 461–481.