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Valuing commodity options and futures options with changing economic conditions

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ABSTRACT

A model for valuing a European-style commodity option and a futures option is discussed with a view to incorporating the impact of changing hidden economic conditions on commodity price dynamics. The proposed model may be thought of as an extension to the Gibson–Schwartz two-factor model, where the model parameters vary when the hidden state of an economy switches. A semi-analytical approach to valuing commodity options and futures options is adopted, where the closed-form expressions for the characteristic functions of the logarithmic commodity price and futures price are derived. A fast Fourier transform (FFT) approach is then applied to provide a practical and efficient way to evaluate the option prices. Real data studies and numerical examples are used to illustrate the practical implementation of the model.

1. Introduction

In recent years, commodity derivatives have become popular in both exchange-traded and over-the-counter markets around the globe. According to a review report by the Bank for International Settlements (2007), it was estimated that the number of outstanding exchange-traded commodity derivative contracts has grown from 12.4 million in June 2003 to 32.1 million in June 2006, more than double in three years. It was also estimated that the growth in the trading activities of commodity derivatives over the same period has been more pronounced with the notional value of outstanding contracts growing from US 1.04 trillion to USD 6.39 trillion, more than five times in three years (see Trolle and Schwartz (2009)). It was noted in Trolle and Schwartz (2009) that a large proportion of commodity derivatives traded in both markets is commodity options. Consequently, the pricing and hedging of commodity derivatives are important issues from the practical perspective.

The main challenge of pricing and hedging commodity options is that one has to take into account the impacts of production and inventory conditions on commodity prices. This makes the pricing and hedging of commodity options more complicated than those of standard stock options. To depict this feature of commodity prices, the concept of convenience yield is introduced. Semi-formally speaking, a convenience yield can be understood as an implied rate of return on inventories. From an investor's perspective, it can be viewed as the benefit when holding a spot commodity rather than its futures contracts. Indeed, one of the key steps in building models for pricing and hedging commodity derivatives is to model the dynamic behavior of a convenience yield. The Gibson–Schwartz two-factor model, introduced by Gibson and Schwartz (1990), is a major contribution along this direction, where a mean-reverting diffusion process is used to model the stochastic evolution of a convenience yield over time. Since the works of Gibson and Schwartz (1990), a considerable amount of attention has been given to the extension of the Gibson–Schwartz two-factor model for pricing commodity futures and options. Schwartz (1997) studied three models to depict the stochastic behavior of commodity prices, by incorporating mean-reversion features, convenience yield and stochastic interest rates. Economic implications and practical applications have also been considered in Schwartz (1997). Casassus and Collin-Dufresne (2005) emphasized a more general situation where the unconditional correlation structure of spot price and convenience yields have been discussed. Paschke and Prokopczuk (2010) developed a new ABM-CARMA (p,q) model and derived closed-form valuation formulae for futures and options. Some other works include Hilliard and Reis (1998), Miltersen and Schwartz (1998), Schwartz and Smith (2000), Nielsen and Schwartz (2004), Nakajima and Ohashi (2011), and Ewald et al. (2015a, 2015b), amongst others.

(Macro)-economic conditions could have significant impacts on commodity prices. An early work which highlights the link between business cycles and commodity prices was attributed to the paper by Fama and French (1988). From a practical perspective, it is of interest to model and investigate the impacts of structural changes in (macro)-economic conditions on commodity prices. Regime-switching models represent a class of econometric models which are widely used in economics and finance to incorporate structural changes in (macro)-economic conditions on economic and financial dynamics,

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particularly asset price dynamics. Hamilton (1989) popularized regime-switching models into financial econometrics. The basic idea of regime-switching models is that one set of model parameters is in force at a time depending on the state of the underlying state process at that time. The state process is usually described by a Markov chain. There have been some works on the use of regime-switching models in describing and investigating commodity prices. For example, Raymond and Rich (1997) adopted a Markov state-switching approach to investigate the relationship between oil prices and (macro)-economic conditions. Haldrup and Nielsen (2006) applied a regime-switching model to describe the long-term memory of electricity prices. Chen and Insley (2012) studied a regime-switching model for stochastic lumber prices with a view to improving the analysis of optimal harvesting problems in forestry. There has been a considerable interest in applying regime-switching models to value options. Some examples are Buffington and Elliott (2002), Ishijima and Kihara (2005), Elliott et al. (2005), Ching et al. (2007), Siu (2008), Yuen and Yang (2009, 2010), Liew and Siu (2010), Elliott and Siu (2012), Liang et al. (2013), Fan et al. (2014), Shen and Siu (2013), and Shen et al. (2014), amongst others. However, it seems that most of the previous works may mainly focus on the valuation of options on equities, while relatively little attention has been given to the valuation of commodity derivatives in regime-switching models. On the other hand, commodity prices seem more vulnerable to structural changes in economic conditions. However, it seems that traditional models for commodity prices such as the Gibson–Schwartz model, may not be able to describe the impacts of structural changes in economic conditions on commodity prices. Consequently, it may be worth investigating the valuation of commodity derivatives in a stochastic convenience yield environment with regime-switching from a practical perspective.

In this paper, we discuss a Markovian regime-switching extension to the Gibson–Schwartz model (MRSGS) for evaluating a European-style commodity option and a futures option taking into account the impacts of structural changes in economic conditions on commodity prices. The key idea is that model parameters such as the mean-reverting level and the volatility of a stochastic convenience yield as well as the volatility of the commodity spot price are modulated by a continuous-time, finite-state, hidden Markov chain. To value the commodity options and futures options, closed-form expressions of the characteristic functions of the logarithmic commodity spot price and the logarithmic futures price are first derived. Then a fast Fourier transform (FFT) approach is used to evaluate both commodity option prices and futures option prices. The FFT approach, introduced by Carr and Madan (1999), provides an efficient method to evaluate the prices of European options due to the faster computation of the discrete Fourier transform (DFT). Some recent works on the use of the FFT approach to price derivative securities in Markovian regime-switching models are, for example, Fan (2013) and Fan et al. (2015) for pricing annuity options, Fan et al. (submitted for publication) for pricing power options, Fan et al. (2014) for foreign equity options, Shen et al. (2014) for options under double regime-switching model, Fan et al. (submitted for publication) for pricing options in a stochastic interest rate environment. Finally, we provide an empirical application of our model using the real data set of West Texas Intermediate (WTI-a light sweet crude oil stream) options. More specifically, we calibrate the model parameters to the market prices of the WTI options and compare the in-sample fitting errors and out-of-sample prediction errors of the Markovian regime-switching Gibson–Schwartz model and the original Gibson–Schwartz model. The empirical analysis illustrates the practical implementation of the model and the impacts of switching regimes on the prices of the commodity options and futures options.

The roadmap of the rest of the paper is as follows. The next section presents the Markovian regime-switching Gibson–Schwartz two-factor model. In Section 3, we first consider the valuation of commodity options and then the valuation of commodity futures options using the FFT approach. Section 4 presents the empirical analysis and numerical examples. The final section provides concluding remarks. The proofs of the lemmas and propositions in the paper are standard and involve the use of standard mathematical techniques, so the proofs are placed in the Appendix A.

2. The model dynamics

In this section, we consider a continuous-time economy with a finite time horizon $\mathcal{T}$, i.e., $\mathcal{T} := [0, T^*]$, where $T^* < \infty$. Let $(\Omega, \mathcal{F}, \mathcal{P})$ be a complete probability space. Following some literature about commodity pricing, we start with a risk-neutral probability measure $\mathcal{P}$.

To describe the evolution of the hidden state of an economy over time, we consider a continuous-time, finite-state, hidden Markov chain $\mathbf{X} := \{\mathbf{X}(t)|t \in \mathcal{T}\}$ defined on $(\Omega, \mathcal{F}, \mathcal{P})$. The states of the chain may represent different states of an economy or different stages of a business cycle, which cannot be directly observed in practical situations. Mathematically, we suppose that the chain $\mathbf{X}$ has the canonical state space $\mathcal{E} := \{\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_N\} \subset \Re^N$, where the j-th component of $\mathbf{e}_i$ is the Kronecker delta δ_{ij}, for each $i, j = 1, 2, \ldots, N$. This canonical state space has been used in Elliott et al. (1995). To specify the probability laws of the chain $\mathbf{X}$, we consider the rate matrix, or generator, $\mathbf{Q} := [q_{ij}]_{i,j=1,2,\cdots,N}$. Then the following semimartingale dynamics for the chain $\mathbf{X}$ was obtained by Elliott et al. (1995):

$$\mathbf{X}(t) = \mathbf{X}(0) + \int_0^t \mathbf{Q}\mathbf{X}(s)ds + \mathbf{M}(t), t \in \mathcal{T}.$$

Here $\{\mathbf{M}(t)|t \in \mathcal{T}\}$ is an $\Re^N$-valued martingale under $\mathcal{P}$ with respect to the filtration generated by $\mathbf{X}$.

We now present a Markovian regime-switching extension to the Gibson–Schwartz two-factor model for both the commodity price and the stochastic convenience yield. Let y' be the transpose of a vector or a matrix y. Denote $\{m(t)|t \in \mathcal{T}\}$ as the mean-reversion level of the stochastic convenience yield process, which represents the long-term convenience yields. We assume that $\{m(t)|t \in \mathcal{T}\}$ changes over time according to the state process of the hidden economy $\{\mathbf{X}(t)|t \in \mathcal{T}\}$ as follows:

$$m(t) := \langle \mathbf{m}, \mathbf{X}(t) \rangle ,$$

where $<\cdot, \cdot>$ is the scalar product in $\Re^N$. Here, $\mathbf{m} := (m_1, m_2, \ldots, m_N)' \in \Re^N$ with $m_i > 0$, for each $i = 1, 2, \ldots, N$. In particular, m_i is the mean-reversion level of the convenience yield process corresponding to the ith state of the hidden economic condition.

Let κ be the parameter controlling the speed of mean reversion for the convenience yield process, where $\kappa > 0$. Define $\{\sigma_S(t)|t \in \mathcal{T}\}$ and $\{\sigma_\delta(t)|t \in \mathcal{T}\}$ as the volatilities of the commodity price and the stochastic convenience yield process, respectively. Again we suppose that these volatilities change over time according to the state process of the hidden economy as follows:

$$\sigma_S(t) := \langle \boldsymbol{\sigma}_s, \mathbf{X}(t) \rangle ,$$
$$\sigma_\delta(t) := \langle \boldsymbol{\sigma}_\delta, \mathbf{X}(t) \rangle ,$$

where $\boldsymbol{\sigma}_S := (\sigma_{S1}, \sigma_{S2}, \ldots, \sigma_{SN})' \in \Re^N$ with $\sigma_{Si} > 0$ and $\boldsymbol{\sigma}_\delta := (\sigma_{\delta 1}, \sigma_{\delta 2}, \ldots, \sigma_{\delta N})' \in \Re^N$ with $\sigma_{\delta i} > 0$, for each $i = 1, 2, \ldots, N$.

To simplify our discussion, we assume that the risk-free rate of interest is a positive constant r. Let $W_S := \{W_S(t)|t \in \mathcal{T}\}$ and $W_\delta := \{W_\delta(t)|t \in \mathcal{T}\}$ be two correlated standard Brownian motions with respect to their right-continuous, $\mathcal{P}$-complete, natural filtrations under $\mathcal{P}$. For the sake of generality, we assume that the instantaneous correlation coefficient of the two Brownian motions changes over time according to the state process $\{\mathbf{X}(t)|t \in \mathcal{T}\}$ as:

$$\rho(t) := \langle \boldsymbol{\rho}, \mathbf{X}(t) \rangle .$$

Here $\boldsymbol{\rho} := (\rho_1, \rho_2, \ldots, \rho_N)' \in \Re^N$ and $-1 < \rho_j < 1$ for $j = 1, \ldots, N$.