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New non-equilibrium cobweb dynamical evolution model and its application

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ABSTRACT

A new cobweb model is built under price adjustment situation. Its price correction function considers the dynamic effects of regulation. Accuracy and authenticity of the analysis are increases. The new model is applied to the analysis of stability of power's supply and demand. An empirical analysis is carried out to study electricity power market in Jiangsu province and Shandong province in China. Results from the new model are not identical with the old one. The new model performs better in showing the dynamic effects with price changes and the dynamic law system. Furthermore, the stability for the new cobweb model without considering external influences is also considered. The study shows the stability of new model by numerical analysis. Probability of stability is given.

1. Introduction

Economic growth and power supply and demand are two important indicators to judge the macroeconomic situation of a country or region. There have been a lot of empirical researches to the relationship between energy consumption and economic growth. The first work is the study of the causual relationship between the power supply demand and macroeconomic variables of U.S. (Kraft and Kraft, 1978).

Glasure and Lee (1998) found bidirectional causality from energy consumption to GDP growth in Singapore by using Granger test. Yu (Yu and Choi, 1985) found causality between South Korea's GDP and energy consumption based on the Granger test. The authors (Alvarado et al., 2001) found the presence of two-way causal relationship between energy consumption and GDP in Asian developing countries based on cointegration test and error correction model. To describe random behavior in power prices, Mari (Linares, 2002) used the transformation system model, which can be used to capture power, forced outage, power shortages during peak demand in a fully competitive market.

Han et al. (2004) researched the relationship between energy consumption and economic growth in China and obtained a two-way causal relationship between energy consumption and economic growth. Wang and Liu (2007) used co-integration analysis and Granger causality test to indicate the unidirectional causality from energy consumption to GDP. Yang (2006) pointed out that the economic growth in Shandong province and energy consumption have the long term equilibrium relationship and there is unidirectional causality from economic growth to energy consumption. Sun (2002) studied factors affecting the perspective of power supply and demand using electricity demand function. Lin analyzed Chinese economic growth and electricity consumption relationships from the power production function (Lin, 2003).

Cobweb theory is the economic theory on the dynamic equilibrium analysis. Starting in the 1930s, it is now used to explain the cycle of fluctuations in certain commodity of prices and production. Assumptions of the cobweb model include: (1) products require a longer period of time, and the supply function in this cycle is pre-priced; (2) there is no time lag in demand, and that demand is a function of the current price; (3) each market is clearing, i.e, the market price is to be adjusted to make the total amount of buyers to absorb the entire supply.

Application of cobweb model in supply-and-demand market is very extensive. Gallas (2014) studied the economic model of the power system when the two variables changed simultaneously. Onozaki (2000) used the cobweb model to show that the faster providers adapt their products, the more inelastic demand, the more chaotic market behaved. Branch (2012) studied the stability condition of the heterogeneous rational expectations in cobweb mode. Commendatore (2008) analyzed a binding cobweb model based on the premise of highly sensitive nature of credit constraint. Westerhoff (2010) considered a cobweb model with factor of technical and fundamental investors' behaviors.

Liu (2003) found both exogenous and endogenous in money supply by using cobweb model. Liu (2005) analyzed farmers' income growth and volatility phenomenon. Zhou (2009) constructed a cobweb model of China's real estate market. Wu (2011) analyzed price fluctuations of hog nearly 10 years based on the cobweb theory. Qu and Cui (2012) studied the relationship between the scale of production and price volatility of corn.

Traditional cobweb models assume that the supply and demand functions are linear and the supply and demand balance in each period.

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But in real economy systems, even the assumption of "equilibrium" is established, it cannot ensure that all of the unstable equilibrium price eventually converge to a stable equilibrium price because its motion is entirely decided by the identified specific form of supply function and demand function. Meanwhile, the reality of the market is unlikely to reach the market clearing installments, thus the total supply and demand is in an imbalance state.

Therefore, markets in a nonequilibrium state should be considered. Basing on non-equilibrium theory, Huang (2004) and Colucci and Valori (2011) introduced regulatory mechanisms in order to achieve market exchange suitable to supply and demand sides. But such researches on non-equilibrium state, with a constant number of adjustable parameters or price adjustment are too simple (see Huang, 2004). They only considered the influence under a constant price adjustment. In fact, the price adjustment is changing, and this is reflected in the rhythm of price fluctuations.

This paper improves to the cobweb model and introduces a new cobweb model. Dynamic evolution of the new model is studied. The new model is applied to the power supply and demand market. Empirical studies demonstrate the merits of the new model.

There are two important differences between the model in this study and previous dynamic cobweb model. First, most previous cobweb model assumes that demand function and supply function are linear, while in this study they can be either linear or nonlinear forms. Second, although conventional non-equilibrium cobweb model may have a constant price adjustment function, the way is too simple to adjust by the same regulation. The model in this study will enhance the price adjustment function, using a function instead of a constant as the price adjustment parameters. Resulting in a variety of non-equilibrium states in the market, our model is an expansion and development of the previous disequilibrium cobweb model.

The rest of this paper consists of four sections. Section 2 introduces the new cobweb model and studies its stability. In Section 3, we study the stability of the new cobweb model when merging two markets. Empirical analysis is carried out in Section 4 by using the cobweb model. The last section is conclusions.

2. New cobweb dynamics evolution model

2.1. The new cobweb model

In the supply and demand market, let the supply and demand function be monotonic in an economic period. Assume that the demand is a function of the current actual price, $D = D(x)$, the supply is a function of the current forecast price x_t, $S = S(x_t)$ and the price adjustment equation is $x_{t+1} = x_t + \alpha(x_t)(F(x_t) - x_t)$. We write them in a system

$$\begin{cases} D = D(x), \\ S = S(x_t), \\ x_{t+1} = x_t + \alpha(x_t)(F_t - x_t), \end{cases} \tag{1}$$

where $F = F(x_t)$ is the current actual price, i.e., $F(x_t) = x = D^{-1}(S(x_t))$, $\alpha_t = \alpha(x_t)(\alpha_t > 0)$ is called price correction function which is a measure of the speed and magnitude of the adjustment to reflect changes in the price along with excess demand. Three equations in the model are the demand equation, the supply equation and the price adjustment equation.

This article assumes that the demand function $D(x)$ is a monotonically decreasing function of the price x, the supply function is a monotonically increasing function of the price x, in other words, the demand increases monotonically as the price decreases and the supply increases monotonically as the price increases. This is consistent with economic significance.

Rewrite the price adjustment equation as

$$x_{t+1} = G(x_t) = x_t + \alpha(x_t)(F(x_t) - x_t).$$

We see that the price correction function is $\alpha(x_t) = \frac{x_{t+1} - x_t}{F(x_t) - x_t}$, i.e., $\alpha(x_t)$ is defined as the ration of the increments of the forecast prices between the current time t and the next time period $t + 1$ over the increments between the current actual price and forecast price. It reflects the changing rate of the forecast prices with the error on price changes. In real life, the difference between the forecast price and the actual price of different time periods fluctuates. If $\alpha(x_t)$ is Frechet derivable at the point x_t (Feil, 2013), that is, the rate of change at the point x_t exists and the derivative at the point x_t changes as x_t changes. If $\alpha(x_t)$ is a constant, the Frechet derivative is constant at the point x_t, which means the rate of change is a certain value and the cobweb model reduces to the old one (Huang, 2004):

$$\begin{cases} D = D(x), \\ S = S(x_t), \\ x_{t+1} = x_t + \alpha(F_t - x_t), \alpha \text{ is a constant.} \end{cases} \tag{2}$$

In the real life, however, the rate of change of the price fluctuates. In rare cases it is a certain value. Hence it is more reasonable to let the price correction function $\alpha_t = \alpha(x_t)$ be a function of the price.

For system (1), we call x^* the equilibrium price of the system if $x^* = G(x^*)$, $x^* \in R^+$. We will analyze the stability of the equilibrium price on supply and demand under different conditions.

In this article, we let $\alpha_t = \alpha_1 + \alpha_2 x_t$, where α_1 and α_2 are undetermined constant.

2.2. Stability analysis of the new cobweb model

We study the disequilibrium cobweb model and its stability when there is only one and n suppliers in the market, respectively. Then, we study the problem when n is an uncertain number. Finally, we study the stability of the market combined by two separate markets.

2.2.1. When there is only one supplier

For a particular market, when there is only one supplier, the equilibrium price of supply and demand system satisfies $x^* = G(x^*)$.

To approach an equilibrium price of supply and demand, the stability condition (Li, 2008) requires $|G'(x^*)| < 1$, that is

$$|1 + \alpha'(F(x^*) - x^*) + \alpha^*(F(x^*) - 1)| < 1.$$

By using the assumption $\alpha(x^*) = \alpha_1 + \alpha_2 x^*$, the above equation is written as

$$-(\alpha_1 + \alpha_2 x^*)F'(x) < 2 - \alpha_1 - \alpha_2(2x^* - F(x^*)).$$

Denote $-\delta = F'(x^*) = \frac{S'(x^*)}{D'(x^*)}$, then the above equation is written as $(\alpha_1 + \alpha_2 x^*)\delta < 2 - \alpha_1 - \alpha_2(2x^* - F(x^*))$, that is $\delta \frac{\alpha_1 + \alpha_2 x^*}{2 - \alpha_1 - \alpha_2(2x^* - F(x^*))} < 1$.

Denote $\beta = \frac{\alpha_1 + \alpha_2 x^*}{2 - \alpha_1 - \alpha_2(2x^* - F(x^*))}$. The inequality is now rewritten as

$$\delta\beta < 1. \tag{3}$$

Here δ is called the degree of instability structure, which is the negative of the ratio of the equilibrium point of supply and rate of change of demand. β is called the degree of erratic behavior, which is a value between 0 and 1 at the equilibrium point.

When $\delta\beta < 1$, i.e., the product of the degrees of instability and erratic behavior of structure is less than 1, the supply and demand stabilize the market. In this case the unstable structure of the market is consistent with the traditional model, remains unchanged. But instability behavior of the market changes. Although the market instability behavior $\beta(x^*) = \frac{\alpha_1 + \alpha_2 x^*}{2 - \alpha_1 - 2\alpha_2 x^* + \alpha_2 F(x^*)}$ changes, where the correction factors α in the former one is replaced by the correction function $(\alpha_1 + \alpha_2 x^*)$, the remaining structure remains unchanged.