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Bilateral counterparty risk valuation for credit default swap in a contagion model using Markov chain

Yinghui Dong [a,b,*], Guojing Wang [c]

[a] Department of Mathematics and Physics, Suzhou University of Science and Technology, Suzhou 215009, PR China
[b] Financial Engineering Research Center, Shanghai Jiao Tong University, Shanghai 200052, PR China
[c] Center for Financial Engineering, Soochow University, Suzhou 215006, PR China

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ABSTRACT

The computation of the bilateral counterparty valuation adjustment for a credit default swap (CDS) contract is in effect the modeling of the default dependence among the investor, the protection seller, and the reference entity. We present a contagion model, where defaults of three parties are all driven by a common continuous-time Markov chain describing the macroeconomic conditions. We give the explicit formula for the bilateral credit valuation adjustment (CVA) of CDS and examine the effect of the regime switching on the CVA.

1. Introduction

After the default of highly rated Lehman Brothers as well as the occurrence of failure in many other large financial institutions, counterparty credit risk has become a hot topic in connection with valuation and risk management of credit derivatives. Counterparty credit risk is the risk that the counterparty to a financial contract will default prior to the expiration of the contract and will not make all the payments required by the contract. Once two counterparties enter into a trade, besides market risk, they also take credit risk against each other. In most cases, the counterparty credit default risk is not considered in direct evaluation of the trades and, therefore, needs to be adjusted appropriately to reflect the risk should either of the counterparties default on their commitments. The adjustment to the value of a default free trading book is what is usually referred to as counterparty valuation adjustment (CVA). For more information on CVA, we refer the interested reader to Gregory (2010) and Cesari et al. (2010). How to value counterparty credit risk in the form of CVA is an active research field, see for example, Brigo and Capponi (2010), Hull and White (2012), and Lipton and Sepp (2009).

Following Brigo and Capponi (2010), in this paper we still investigate the valuation of CVA for CDS. To consider the bilateral CVA for CDS, the most important point is to model the default dependence among the investor, the protection seller and the reference entity. We are therefore interested in developing models and methods which can lead to computable, or even explicit formulas for the valuation of CVA for CDS.

The contagion models are some of the most important models and have been studied quite extensively in recent years. Under the framework of contagious defaults, the default risk is modeled by the reduced-form approach, where the probability of default is determined by an exogenously specified instantaneous default intensity. The contagious defaults are affected by an inter-dependent default risk structure between the parties, where the default intensity of one party increases when the default of another party occurs. See, for example, Jarrow and Yu (2001) create the default contagion effect by introducing a positive jump in the default intensity whenever there is an occurrence of a default of a counterparty. Solving a contagion model faces an obstacle of looping default problem. Bao et al. (2012) construct a particular

* Corresponding author.
E-mail addresses: dongyinghui1030@163.com (Y. Dong), wangguojing@hotmail.com (G. Wang).

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contagion model with stochastic intensities and they use a survival measure approach, proposed by Collin-Dufresne et al. (2004), to give a semi-analytical solution for unilateral CVA. Leung and Kwok (2009) present a continuous-time Markov approach in dealing with contagion models. Motivated by them, in this paper, we consider a contagion model under a Markov environment, and we aim at deriving a closed-form expression for bilateral CVA of CDS.

As we know, credit derivatives are long term instruments and thus it is important to take into account the cyclical effects of the market. These effects can be captured using an appropriate continuous time Markov chain modeling the regime switching economy; see, for example, Graziano and Rogers (2009) who assume that there exists a continuous-time finite-state irreducible Markov chain, which drives the common dynamics of the credit in the portfolio. Their model allows us to obtain a closed-form expression for the joint and marginal default probabilities. In this paper, extending Graziano and Rogers (2009), we incorporate contagion into the reduced-form model under a Markov environment. It is well known that it is very difficult for us to give the explicit formula for the credit valuation adjustment under a general contagion model. But under the contagion model with regime-switching intensities we propose, the closed-form formula for the CVA can be derived based on the explicit expression for Laplace transform of the regime-switching process.

Indeed, regime-switching models, introduced by Hamilton (1998), have gained immense popularity in the finance and insurance domain. In a regime-switching model, the market is assumed to be in different states depending on the state of the economy. Regime shifts from one economic state to another may occur due to various financial factors like changes in business conditions, management decisions and other macroeconomic conditions. Empirical studies point to the existence of different regimes in the default risk valuation, see Davies (2004) and Giesecke et al. (2011). Regimes appearing in the default-intensity functions can result in a large degree of flexibility in the model specifications. Many papers point out that incorporating the impact of macroeconomic conditions and business cycles via the introduction of the Markovian regime switching effect has important empirical implications and explains some important empirical behaviors of observed options price and credit spreads, see, for example, Buffington and Elliott (2002), Hackbarth et al. (2006) and Shen and Siu (2013).

Therefore, we incorporate regime switching and contagion into the modeling of the default dependence. Under the framework of the contagion model with regime switching, we can derive a closed-form expression for the bilateral CVA of CDS. The paper is organized as follows: Section 2 describes the cash flows of a payer CDS with and without counterparty credit risk, and further gives a formula for the bilateral CVA of this CDS in a general set-up. Section 3 introduces the contagion model under the regime-switching framework, which forms the basic building blocks for the valuation of the bilateral CVA for CDS. The closed-form formula for the bilateral CVA is presented in Section 4. Section 5 gives some numerical results. Section 6 concludes.

2. Bilateral credit valuation adjustment

In this Section, we give a formula for the bilateral CVA of a CDS in a general framework. Given a filtered complete probability space $\{\Omega, \Im, \{\Im_t\}_{0\le t\le T}, P\}$ all random variables of this paper are assumed to be defined on it. Let E_τ stand for the conditional expectation under P given $\Im_\tau$, for any stopping time τ.

Denote by $D(t,T)$ the price of a zero-coupon bond with maturity T at time t. Consider a CDS contract with notional value one, continuous spread rate payments κ and maturity T. Indices 1, 2, and 3 refer to quantities related to the investor, the reference entity and the counterparty. Denote by τ_1,τ_2 and τ_3 the default times of the investor, the reference entity and the counterparty, respectively; denote respectively by R_1, R_2 and R_3 the recoveries of the investor, the reference entity and the counterparty, supposed to be constant. In this paper, assume that all the cash flows and prices are considered from the perspective of the investor and that there are no simultaneous defaults. Denote by $x^+ = \max\{x,0\}$ and $x^- = -\max\{x,0\}$ be the positive part and the negative part of $x, x\in \mathbf{R}$, respectively. Firstly, we give the discounted cash flows of CDS with and without counterparty credit risk.

Definition 2.1. The model price process of a risk-free CDS is given by $P_t = E_t[p_T(t)]$, where $p_T(t)$ corresponds to the risk-free CDS cumulative discounted cash flows on the time interval $(t,T]$, so

$$p_T(t) = -\kappa\int_{t\wedge\tau_2\wedge T}^{T\wedge\tau_2} D(t,s)ds + (1-R_2)D(t,\tau_2)1_{\{t<\tau_2\le T\}}, \tag{2.1}$$

with $p_T(t) = 0$ for $t \ge \tau_2 \wedge T$.

Now let us turn to the model price process of a risky CDS. Define $\overline{\tau} = min\{\tau_1, \tau_3\}$. If $\overline{\tau} > T$, there is neither a default of the investor, nor a default of his counterparty during the life of the CDS contract. On the contrary, if $\overline{\tau} \le T$, then a fair value of the CDS is computed at time $\overline{\tau}$. In this paper, we specify that the fair value at $\overline{\tau}$ is the value at time $\overline{\tau}$ of a risk-free CDS on the same reference name $P_{\overline{\tau}}$. Note that there are no simultaneous defaults. Then we distinguish two cases:

1. $\overline{\tau} = \tau_3 < \tau_1$. If P_{τ_3} is negative for the investor, it is completely paid by the investor. If P_{τ_3} is positive for the investor, the counterparty is assumed to pay only a recovery fraction R_3 of P_{τ_3} to the investor.
2. $\overline{\tau} = \tau_1 < \tau_3$. If P_{τ_1} is negative for the defaulted investor, only a recovery fraction R_1 of P_{τ_1} is paid by the investor to the counterparty. If P_{τ_1} is positive for the investor, it is completely paid by the counterparty.

Therefore, from the above description, we have

Definition 2.2. The model price process of a risky CDS is given by $\prod_t = E_t[\pi_T(t)]$, where $\pi_T(t)$ corresponds to the risky CDS cumulative discounted cash flows on the time interval $(t,T]$, so

$$\begin{aligned}\pi_T(t) = &-\kappa\int_{\tau_1\wedge\tau_2\wedge\tau_3\wedge t}^{T\wedge\tau_1\wedge\tau_2\wedge\tau_3} D(t,s)ds + D(t,\tau_2)(1-R_2)1_{\{t<\tau_2\le T\}}1_{\{\tau_2<\tau_1\wedge\tau_3\}}\\ &+D(t,\tau_1)1_{\{t<\tau_1\le T,\tau_1<\tau_2\wedge\tau_3\}}\left(P_{\tau_1}^+ - R_1P_{\tau_1}^-\right)\\ &+D(t,\tau_3)1_{\{t<\tau_3\le T,\tau_3<\tau_1\wedge\tau_2\}}\left(R_3P_{\tau_3}^+ - P_{\tau_3}^-\right),\end{aligned} \tag{2.2}$$

with $\pi_T(t) = 0$ for $t \ge \tau_1\wedge\tau_2\wedge\tau_3\wedge T$.

The credit valuation adjustment (CVA) is the difference between the risk-free portfolio value and the true portfolio value that takes into account the possibility of a counterparty's default. In other words, CVA is the market value of counterparty credit default risk. Then the formula for the bilateral CVA of a CDS contract is given by

Proposition 2.1. At valuation time t, and conditional on the event $\{\tau_1\wedge\tau_2\wedge\tau_3 > t\}$, we have

$$\begin{aligned}CVA_t = &E_t\left[D(t,\tau_3)(1-R_3)P_{\tau_3}^+1_{\{\tau_3<\tau_1\wedge\tau_2, t<\tau_3\le T\}}\right]\\ &-E_t\left[D(t,\tau_1)(1-R_1)P_{\tau_1}^-1_{\{\tau_1<\tau_2\wedge\tau_3, t<\tau_1\le T\}}\right].\end{aligned} \tag{2.3}$$

Proof. If $\tau_2 \le t$ or $\tau_1\wedge\tau_2\wedge\tau_3 > T$ then it follows from Definitions 2.1, 2.2 that $CVA_t = 0$ Assume $T \ge \tau_1\wedge\tau_2\wedge\tau_3 > t$, and we can divide the event $\{T \ge \tau_1\wedge\tau_2\wedge\tau_3 > t\}$ into three mutually exclusive events:
$A = \{\tau_2 < \tau_1\wedge\tau_3, t < \tau_2 \le T\}, B = \{\tau_1 < \tau_2\wedge\tau_3, t < \tau_1 \le T\}$,
$C = \{\tau_3 < \tau_1\wedge\tau_2, t < \tau_3 \le T\}$.

Then under the assumption $T \ge \tau_1\wedge\tau_2\wedge\tau_3 > t$,

$$\begin{aligned}p_T(t) - \pi_T(t) = &(1-R_2)D(t,\tau_2)1_{\{t<\tau_2\le T\}}(1_A + 1_B + 1_C) - \kappa\int_{\tau_1\wedge\tau_2\wedge\tau_3}^{\tau_2\wedge T} D(t,s)ds\\ &-D(t,\tau_2)(1-R_2)1_A - D(t,\tau_1)1_B\left(P_{\tau_1}^+ - R_1P_{\tau_1}^-\right) - D(t,\tau_3)1_C\left(R_3P_{\tau_3}^+ - P_{\tau_3}^-\right).\end{aligned} \tag{2.4}$$