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Equity portfolio insurance against a benchmark: Setting, replication and optimality

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ABSTRACT

This paper undertakes the issue of portfolio insurance from the perspective of a risk-averse agent requiring his financial wealth to grow at a floored rate in excess of an equity benchmark. The suggested solution is a generalization of the CPPI approach within a two-equity asset framework. The paper examines some features of this extension related to its dynamic, its relative risk–reward profile and its static replication. It focuses more specifically on the optimal design of this portfolio strategy in the sense of consumption–investment decision making.

1. Introduction

Portfolio insurance strategies, including constant proportion portfolio insurance (CPPI), have gained significant traction in the asset management industry over the last decade. Essentially these stop-loss strategies are designed in such a way that allows the investor to recover a predefined proportion of the initial capital at expiry. An asymmetric return profile is therefore yielded. This feature turns out to be particularly relevant to an investor whose wealth is linked to a risk-free asset.

CPPI was first introduced by Black and Jones (1987) and implemented through equity securities, then Perold and Sharpe (1988) extended the approach to fixed income assets. It has been extensively studied in the literature since then, and enhanced in many ways. Black and Perold (1992) provided a general theoretical framework to account for transaction costs along with borrowing limits. Bookstaber and Langsam (2000), followed by Bertrand and Prigent (2005), ran comparative analyses between CPPI and option based portfolio insurance (OBPI). Bertrand and Prigent (2003) extended this analysis to take on stochastic volatilities, while Cont and Tankov (2009) studied the behavior of this approach under models where asset prices may experience downward movements. A similar issue was explored earlier by Bertrand and Prigent (2002) using the extreme value theory. The CPPI sophistication literature also includes Prigent and Tahar (2005) who introduced a new CPPI variant with an embedded cushion protection in order to overcome the market V-scenario problem (also referred to as the cash lock-in problem).

Although the CPPI approach was originally introduced within a framework consisting of a risky reserve asset (Black and Jones, 1987; Black and Perold, 1992), very little attention has been paid to the issue of investors who require a relative performance insurance against risky benchmarks as opposed to capital insurance.[2] At first sight, the solution to this issue relies on a natural extension of CPPI to the case where the value of the reserve asset is permanently subject to exogenous risks. Amenc et al. (2004) have heuristically suggested such an extension within an enhanced core-satellite allocation approach. They empirically showed that this dynamic approach allows for an asymmetric management of the portfolio tracking error. In a 2006 paper the authors provided a formal characterization of the portfolio cushion dynamic assuming the reserve asset is comprised of risky bonds. The dynamics issue has been also tackled by Bertrand et al. (2010) for a comparable

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[2] A closely related strand of the literature tackled the broad problem of optimal investment strategies for a benchmarked risk-averse investor without explicit insurance requirements. This literature includes Carpenter (1995) who assessed the utility-maximizing portfolio strategy from the point of view of an unconstrained portfolio manager whose wealth is linked to a benchmark through non-hedged incentive fees. Browne (1999) found, under market incompleteness, a range of unconstrained constant mix strategies that are optimal with respect to a variety of criteria including: maximizing the probability of beating the benchmark by a given percentage, minimizing the expected time until the investor beats the benchmark, maximizing the expected discounted reward of outperforming, as well as minimizing the discounted penalty paid upon being outperformed.

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unrestricted finite-horizon extension of CPPI. They relied on analytical expressions of the payoff distribution moments to compare this strategy to a static option-based approach. Moreover, this generalization of CPPI bears also a close resemblance to the so-called liability driven portfolio insurance (LDPI) strategies. This family has been deeply analyzed by Lindset (2004) within the context of pension schemes and life insurance contracts. Although the principles are the same (i.e. protect excess returns over a benchmark), the latter deals with special cases where the liability value is mainly driven by interest rates and inflation.

Even if a formal extension of CPPI to a two-risky asset framework has been more or less discussed in the aforementioned papers, its optimal setting in the sense of consumption–investment decision making is left unexamined to our knowledge. This paper intends to bridge this gap. I restated the portfolio insurance problem within a two-equity asset (a benchmark and an alternative asset) frictionless economy from the perspective of a risk-averse agent requiring financial wealth to grow at a floored rate in excess of an equity benchmark. I then assigned a formal characterization of the appropriate constrained portfolio insurance strategy in this respect. This strategy is labeled benchmark-driven portfolio insurance (BDPI) in the remainder of the paper. I also draw some general conclusions regarding the dynamics and the risk-reward patterns of BDPI. Moreover, I showed that the open-ended variant of BDPI (i.e. infinite holding period) can be replicated through a purchase of perpetual American exchange options. The option-based replicating portfolio allows for a static hedging of the strategy. It also allows overcoming the path-dependency issue in order to deal with the optimality of BDPI. So I relied on Black's and Perold's (1992) idea of introducing dividend consumption once the replicating options are exercised in order to remove the path-dependency due to the exposure constraint and, therefore, render BDPI suitable for utility-maximization. This enables us to find the set of optimal conditions for a general form of piecewise utility functions with a minimum consumption constraint. I proved that consumption is only prevailing and increasing in wealth when the exposure limit is binding and that the optimal strategy is BDPI set according to the shape of the utility function. Specifically, when the piecewise iso-elastic utility function class is considered, then the size of the investment multiple is increasing with the agent's impatience to consume. In this case consumption becomes linear above the wealth level at which the exposure limit is reached. The optimal limit depends on the agent's relative risk aversion. In addition, the multiple rises with the alternative asset information ratio when the latter is large enough to offset the effect of the impatience to consume. Optimal consumption decreases with the information ratio at a pace that depends on the agent's relative risk aversion.

The practical implication of this paper is twofold. First, it underscores the underperformance risk issue for investors with benchmarked investment policies. Practically, when it comes more specifically to benchmarked equity portfolio allocation, BDPI would be more accurate than mainstream tactical methods bordered by deviation risk budgets (i.e. tracking error targets), which are still the most widely used by the profession. The intuitive reason behind is when dealing with asset classes that require looking beyond the second moment of excess returns, dynamic allocations based on a formal risk budgeting, and handling endogenously the down side risk as in BDPI, turn out to be more robust than approaches exclusively driven by tracking error constraints. Second, our work sets a practical framework for undertaking preliminary analyses of portfolio insurance in the context of benchmarked equity portfolio management. It therefore provides many potential patterns of enhancement in order to concretely put BDPI into application.

This paper proceeds as follows. First, a short description of BDPI is provided along with the basic notations used throughout. The dynamics and the option-based replication issues are brought up subsequently. The last section undertakes the issue of the optimal setting rule in the sense of consumption–investment decision making.

2. The BDPI approach: assumptions and basic notations

This section aims to concisely describe the basics of the BDPI approach, make explicit the main underlying assumptions and set the notations that will be used in the remainder of the paper.

2.1. Assumptions and basic notations

Assume a frictionless economy set at time $t = 0$ with two liquid equity assets: a reference asset (RA) and an active asset (AA) whose prices at any time $t \geq 0$ are denoted by $S_1(t)$ and $S_2(t)$ respectively. Both assets are payout-protected in that all the cash flows are assumed to be reinvested in the asset that yielded them (i.e. total return assets). Unless otherwise stated,[3] the joint dynamic of the two assets prices under the conditional historical probability Q_t are given by the following two-dimensional Ito process:

$$\frac{dS_1(t)}{S_1(t)} = \mu_1(t)dt + \sigma_1(t)dW_1(t) \tag{1.1}$$

$$\frac{dS_2(t)}{S_2(t)} = \mu_2(t)dt + \sigma_2(t)dW_2(t), \tag{1.2}$$

where $W_{i\in\{1,2\}}(t)$ are correlated Brownians under Q_t (with $dW_1(t)dW_2(t) = \rho(t)dt$), $\mu_{i\in\{1,2\}}(t)$ and $\sigma_{i\in\{1,2\}}(t)$ are adapted processes denoting respectively the means and the standard deviations of the assets instantaneous returns such as:

$$E_{Q_t}\left(\int_0^t \left|\mu_{i\in\{1,2\}}(s)\right| ds\right) < \infty \quad \text{and} \quad E_{Q_t}\left(\int_0^t \sigma^2_{i\in\{1,2\}}(s) ds\right) < \infty.$$

Let's consider in addition that portfolio choices are assessed from the perspective of a representative risk-averse investor, who has an investment horizon $T > t$ and whose financial wealth is indexed to RA. Therefore, he assesses the utility from financial outcomes relative to RA. Put differently, the investor's financial wealth reserve is valued based on the RA-numeraire. He expects AA to deliver higher return than RA on average,[4] which implies:

$$\hat{\mu}_2 > \hat{\mu}_1, \quad \text{with} \quad \hat{\mu}_{i\in\{1,2\}} = \frac{1}{T}\int_0^T \mu_{i\in\{1,2\}}(s)ds. \tag{1.3}$$

Moreover, the representative investor is assumed to require a loss-limit on his portfolio over the investment horizon set as a proportion α (or a gearing) of RA total return. With respect to this constraint, the portfolio value $V(t)$ has a floor referred to by $F(t)$, with:

$$F(t) = (1-\alpha)\frac{S_1(t)}{S_1(0)}V(0). \tag{2.1}$$

The excess of the portfolio value over the floor is called the cushion $C(t)$, with:

$$C(t) = V(t) - F(t). \tag{2.2}$$

[3] Note that for practical reasons we will need to make later the much stronger assumption that the assets prices follow a geometric Brownian motion (GBM hereafter), which is a special case of the dynamic in Eqs. (1.1) and (1.2).

[4] Broadly, the expectations on the dynamic parameters (i.e. volatilities, drifts and correlation) represent the investor's *ex-ante* view on AA an RA joint dynamic.