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The virtue of overconfidence when you are not perfectly informed

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ABSTRACT

Based on Foster and Viswanathan (1994), this work investigates how the heterogeneous beliefs affect equilibrium results when agents are informed asymmetrically. We find that the equilibrium remains the same whether or not the better informed agent is overconfident, but it is a virtue for the less informed agent to be overconfident since this helps him survive in competing with the other less informed agents, however, the less informed agents cannot earn more than the better informed agent, no matter how overconfident he is.

1. Introduction

Based on Foster and Viswanathan (1994), this work characterizes the optimal strategies of asymmetrically informed traders each endowed with overconfident beliefs. Kyle (1985) first investigates the optimal strategies of an agent who possesses a private signal of the risky asset's fundamental value. Following Kyle, Holden and Subrahmanyam (1992) model the competition among identically informed agents. Foster and Viswanathan (1994), Zhang (2008) and Liu and Zhang (2011) examine the competition among asymmetrically informed agents.[1] Since abundant evidence shows that most people most of the time are overconfident in the sense that they overestimate the precision of their knowledge, Kyle and Wang (1997) characterize how the overconfident insiders compete when they are informed to the same extent. Naturally, it is important and interesting to see how overconfident insiders behave when they are informed to different extent.

As Kyle and Wang (1997) show, when insiders are symmetrically informed, an overconfident insider earns more than his rational competitor since the overconfidence belief plays as a commitment to trade aggressively. It is interesting to raise the following questions in our model with asymmetrically informed insiders. (i) Can the overconfidence belief help the less informed agent earn more than the better informed agent? (ii) Does the better informed agent's belief plays the same role in equilibrium as the less informed agent' belief? (iii) How the overconfidence belief affects the price efficiency in transmitting information among traders?

The main findings of our model are as follows. (i) The overconfident imperfectly informed agent earns more than his rational imperfectly informed competitor, but he cannot earn more than the perfectly informed competitor. (ii) The imperfectly informed agent's belief can affect the equilibrium results but the perfectly informed agent's belief cannot affect them. (iii) Under certain conditions, the imperfectly informed agents earn more when they are overconfident than when they are rational. (iv) Enhancing the imperfectly informed agent's confidence degree leads to a higher trading intensity on both the perfect and imperfect information, and thus it yields a higher amount of information transmitted from the better informed agent to the less informed agents and to the uninformed agents.

In Section 2 we discuss the structure of our model. Section 3 derives the linear equilibrium and presents the properties of our model's equilibrium results. Section 4 concludes.

2. The model

A risky asset's fundamental value $\tilde{v}$ is normally distributed with prior mean p_0 and prior variance σ_v^2, in statistical notation, $v \sim \mathcal{N}(p_0, \sigma_v^2)$. An imperfect signal concerning the fundamental value, denoted as $\tilde{s}$, satisfies

$$\tilde{s} = \tilde{v} + \tilde{\varepsilon}$$

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[1] Foster and Viswanathan (1996) and Cao and Ma (2000) also study the competition among differently informed agents. Foster and Viswanathan (1994), Zhang (2008) and Liu and Zhang (2011) differ from them in that the better informed agent's information subsums the less informed agents', whereas in Foster and Viswanathan (1996) and Cao and Ma (2000), there's no such setting.

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in which $\tilde{\varepsilon} \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right)$ and $\tilde{\varepsilon}$ is independent of $\tilde{v}$. There are four kinds of traders: a better (perfectly) informed trader observing the realization of signal $\tilde{s}$ as well as the actual value of the asset $\tilde{v}$, N less (imperfectly) informed traders observing only $\tilde{s}$,[2] a representative market maker and multiple noise traders having no information prior to the trading.

In our model, both the better and the less informed traders can be overconfident. Specifically speaking, the better informed agent, denoted as insider 0, and the less informed trader, denoted as insider $i(i = 1,..., N)$, believe respectively that

$$\tilde{s} = \tilde{v} + \kappa_i \tilde{\varepsilon}, \quad i = 0, 1, \ldots, N$$

where $\kappa_i \le 1$ and κ_i^{-1} represents the confidence degree of the ith informed trader. The market maker and noise traders are rational. In our model, traders have different estimates about the asset's fundamental value, and they "agree to disagreement" as that in the seminal paper of Harrison and Kreps (1978).[3]

In the trading, each insider submits an order to maximize his expected profits based on the information available and the belief mentioned above. Denote the better informed agent's order $\tilde{x}_b$, and the ith$(i = 1,..., N)$ less informed agent's order $\tilde{x}_{l_i}$. Meanwhile, noise traders together submit an exogenous order $\tilde{u} \sim \mathcal{N}\left(0, \sigma_u^2\right)$. Assume that $\tilde{u}, \tilde{v}, \tilde{\varepsilon}$ are mutually independent. The representative market maker observes the total order flow $\tilde{y} = \tilde{x}_b + \sum_{i=1}^N \tilde{x}_{l_i} + \tilde{u}$, but he cannot observe $\tilde{x}_b, \tilde{x}_{l_i}, \tilde{u}$ or any other composition of $\tilde{y}$ individually, and he sets the price as

$$\tilde{p} = E[\tilde{v}|\tilde{y}].$$

Use π_b, π_{l_i} to denote respectively the better informed trader's profits and the ith less informed trader's profits. Use $E(\cdot|\cdot)$ and $E_{\kappa_i}(\cdot|\cdot)$ to denote respectively the conditional expectation under the rational belief and that under the ith informed trader's belief.

3. The equilibrium

In a linear equilibrium, all the insiders and the market maker postulate a linear relationship between the price fluctuation and the submitted total order:

$$\tilde{p} = \lambda \tilde{y} + p_0, \tag{1}$$

where the liquidity parameter λ is a constant and its value is announced by the market maker before all traders make their trading decisions.

In equilibrium, the ith less informed trader evaluates the asset's fundamental value $\tilde{v}$ as

$$E_{\kappa_i}(\tilde{v}|\tilde{s}) = c_{\kappa_i}\tilde{s}, \quad \text{with} \quad c_{\kappa_i} = \frac{\sigma_v^2}{\sigma_v^2 + \kappa_i^2 \sigma_\varepsilon^2}.$$

The equilibrium results are given by Theorem 1.

Theorem 1. *Given the pricing strategy of the market maker (1) and the following trading strategies of insiders*

$$\tilde{x}_b = \beta_b(\tilde{v} - p_0) + \alpha_b(\tilde{s} - p_0), \quad \tilde{x}_{l_i} = \alpha_{l_i}(\tilde{s} - p_0), \quad i = 1, \cdots, N, \tag{2}$$

for $\kappa_1,..., \kappa_N$ satisfying $\sum_{n=1}^N c_{\kappa_n} < (N+2)c_1^{1/2}$,[4] there exists a Nash equilibrium, which is unaffected by the better informed agent's confidence degree and is characterized by the following constants:

$$\lambda = \frac{1}{2\sigma_u}\sqrt{\sigma_v^2 - \left(\frac{\sum_{n=1}^N c_{\kappa_n}}{N+2}\right)^2 (\sigma_v^2 + \sigma_\varepsilon^2)}, \tag{3}$$

$$\beta_b = \frac{1}{2\lambda}, \tag{4}$$

$$\alpha_b = -\frac{\sum_{n=1}^N c_{\kappa_n}}{2(N+2)\lambda}, \tag{5}$$

$$\alpha_{l_i} = \frac{1}{2\lambda}\left[c_{\kappa_i} - \frac{\sum_{n=1}^N c_{\kappa_n}}{N+2}\right], \quad i = 1, \cdots, N. \tag{6}$$

Proof. See Appendix A. □

Theorem 1 tells us that varying the less informed agents' beliefs on $\tilde{s}$ affects the equilibrium results, while varying the better informed agent's belief has no effect on equilibrium. In fact, the less informed agents have to estimate the precise information $\tilde{v}$ from the noisy information $\tilde{s}$ when evaluating expected profits, while the better informed agent does not have to do this, and hence his belief about the distribution of $\tilde{s}$ does not affect equilibrium.

From Theorem 1, the better informed trader trades positively on the private information but negatively on the information known by the opponent. This result is similar to that in Luo (2001) where the insider trades positively on the private information but negatively on the publicly known information.

For a less informed agent, Eq. (6) shows that he would trade positively on his information only when his estimation is above that of the "average" level, i.e., $c_{\kappa_i} > \sum_{j=1}^N c_{\kappa_j}/(N+2)$. When $c_{\kappa_i} < \sum_{j=1}^N c_{\kappa_j}/(N+2)$, for a buy signal, the ith less informed agent chooses to sell because he thinks that the other informed agents overbuy the asset and hence the price that would prevail if he also buys is too high. This is in sharp contrast to that in Foster and Viswanathan (1994) where the less informed agent always trades positively on his signal.

When $N = 0$, we have

$$\lambda = \frac{\sigma_v}{2\sigma_u}, \quad \beta = \frac{1}{2\lambda}, \quad \alpha_b = 0.$$

This is exactly the Kyle's (1985) result.

When $\sigma_\varepsilon = 0$, we have

$$\lambda = \frac{\sqrt{N+1}}{N+2}\frac{\sigma_v}{\sigma_u}, \quad \beta_b + \alpha_b = \alpha_{l_n} = \frac{1}{(N+2)\lambda}, \quad n = 1, \ldots, N.$$

[2] The better informed trader represents the agent who possesses a big information advantage such as the financial institute, while the less informed traders might be those agents with some access to a piece of information from the better informed trader.

[3] The confidence degree is exogenous in our model, for the endogenous confidence degree, a dynamic framework is needed to characterize the evolution of belief, such as that in Gervais and Odean (2001).

[4] It means that for a precise enough signal, i.e., $\sigma_\varepsilon^2 < \left(\frac{(N+2)^2}{N^2} - 1\right)\sigma_v^2$, each insider can be arbitrarily irrational, but for a noisy signal, i.e., $\sigma_\varepsilon^2 \ge \left(\frac{(N+2)^2}{N^2} - 1\right)\sigma_v^2$, the insiders must be moderately overconfident, that is, κ_n, $n = 1 \ldots N$ must satisfy $\sum_{n=1}^N \frac{1}{\sigma_v^2 + k_n^2\sigma_\varepsilon^2} \le \frac{N+2}{\sqrt{(\sigma_v^2 + \sigma_\varepsilon^2)}\sigma_v}$.