Economic Modelling 47 (2015) 193–206

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

Decomposing and valuing convertible bonds: A new method based on exotic options

Yun Feng [a,*], Bing-hua Huang [a], Martin Young [b], Qi-yuan Zhou [c]

[a] Antai College of Economics & Management, Shanghai Jiao Tong University, 535 Fa Hua Zhen Road, Shanghai 200052, China
[b] Massey University, New Zealand
[c] Zhonghai Fund Management Co., Ltd., China

ARTICLE INFO

Article history:
Accepted 7 February 2015
Available online 12 March 2015

Keywords:
Convertible bonds
Complete decomposition
Exotic options
Call provisions
Put provisions

ABSTRACT

We use exotic options to develop a complete decomposition method for analyzing callable convertible bonds (CCBs), and puttable callable convertible bonds (PCCBs) with credit risk. Since exotic options are path-dependent while vanilla options are not, exotic options can better address the risk exposure, and better replicate the payoff features of CCBs and PCCBs embedded with path-dependent options, than do vanilla options or warrants used by previous decomposition methods. Our method provides investors with an effective tool to analyze the effects and interactions of the different provisions contained in CCBs and PCCBs. This provides better insight into the valuation and analysis of CCBs and PCCBs.

1. Introduction

We propose a complete decomposition method for valuing callable convertible bonds (hereafter CCBs) and puttable callable convertible bonds (hereafter PCCBs) by employing exotic options. Based on this method, we show that a coupon-bearing CCBs and PCCBs can be completely decomposed into its corresponding ordinary bond (with the same principal, coupons and maturity) and several kinds of exotic options without error. The advantages of this approach are significant.

Until now there has been no method presented in the literature to decompose accurately the CCBs and PCCBs into simple tradable securities, though simulation and formulae based on vanilla options have been employed in the valuation process. These existing decomposition methods, mainly based on vanilla options, cannot replicate the feature of payoffs of CCBs and PCCBs completely, resulting in significant pricing errors. In the recent literature, simulation methods are usually the favorite choice for pricing path dependent derivatives like convertible bonds, hereafter CBs (Ammann et al., 2008; Batten et al., 2014). Besides, lattice-based numerical methods which include finite-difference method and binomial/trinomial trees are widely used (Anderson and Buffum, 2004; Ayache et al., 2003; Carayannopoulos and Kalimipalli, 2003; Hung and Wang, 2002; Tsiveriotis and Fernandes, 1998). However, first, although the lattice-based methods have the privilege of high computation efficiency, it cannot deal with path-dependent problem completely. Second, both simulation methods and lattice-based methods fail to demonstrate intuitively and explicitly the values of, and the interactions among the provisions contained in CCBs or PCCBs since they do not provide analytical formulae. These problems are impediments to hedging convertible bonds in practical applications. Third, dealing with most path dependent features can become rather complex given the numerical procedures required. To solve these problems, it is necessary to find a way to completely decompose convertible bonds into several tradable securities, which can each be priced by an analytic formula.

To address this problem, the literature has been devoted to developing appropriate analytical formulae. In terms of the development of the literature in this area, Ingersoll (1977) and Brennan and Schwartz (1977) pioneered the use of contingent claim models to price CBs. The valuation models of CBs are usually divided into two distinct frameworks, the firm-value-based approach and the equity-based approach. Ingersoll (1977), under restrictive assumptions, proved that a non-callable convertible bond is equal in value to its corresponding ordinary bond plus a warrant, and derived the analytic valuation formula for this. He also proved that a callable convertible discount bond is equal in value to its corresponding ordinary discount bond plus a call warrant minus an additional term representing the negative value of the call privilege to the equity owners. Nyborg (1996) extended Ingersoll's work and provided conditions for which there is a closed form solution for the value of CBs when there is senior debt and the common stock

* Corresponding author. Tel.: +86 21 52301390.
E-mail addresses: fengyun@sjtu.edu.cn (Y. Feng), huda608@sjtu.edu.cn (B. Huang), M.Young@massey.ac.nz (M. Young), zhouqy@zhfund.com (Q. Zhou).

http://dx.doi.org/10.1016/j.econmod.2015.02.005
0264-9993/

pays dividends. However, the models of both Ingersoll and Nyborg are firm-value-based models which have difficulty in estimating firm value and firm volatility since not all of the firm's assets are tradable. By adopting the equity-based model, Connolly (1998, chapter 8) decomposed a non-callable convertible bond into its corresponding ordinary bond and a European call warrant. Similarly, Ho and Pfeffer (1996) proposed that a callable convertible bond was equal in value to the value of its corresponding ordinary bond plus the value of an embedded warrant, minus the value of the forced conversion. However, they could not provide the analytic valuation formula. It is worth noting that the forced conversion value demonstrated in their model actually reflects the interactions between the embedded options.

Generally, these formulae, based on vanilla options, do not consider the interactions between the conversion provision, the call provision and the put provision sufficiently and this leads to significant pricing errors (see Section 3). In fact, due to these interactions, the value relating to these three embedded options are both path-dependent. Moreover, the exercise time of these three options is indeterminate. When CCBs or PCCBs is decomposed by using plain vanilla options or warrants, there will be biases in the pricing results because the vanilla options are not path-dependent (Ho and Pfeffer, 1996). Our methods find a solution to these problems.

The method developed in our paper provides three clear and important contributions to the literature. First, we provide an accurate pricing formula for the CCBs or PCCBs since we take into account the interactions between the embedded options in the CCBs or PCCBs. Numerical experiments validate the accuracy of our method. Second, our pricing formulae show both intuitively and explicitly the value of each embedded feature in the CCBs or PCCBs. The financial analyst, the investor and the issuer can conduct quantitative analysis on the change in the value of a CCB or PCCB when some provision, such as the coupon clause, is added to or removed from a contract, or when considering credit risk in the pricing of a CCB or PCCB. Finally, the accuracy of the pricing model in our paper helps the investor and the financial institution identify arbitrage opportunities that may exist between the CBs markets and other financial markets.

The remainder of this paper is organized as follows. Section 2 analyzes theoretical values of a CCB in terms of optimal call policies and optimal conversion strategies under stated assumptions. Section 3 discusses two typical decomposition methods based on vanilla options and then compares the solutions by Monte Carlo simulation with those of the two decomposition methods. Section 4 presents our complete decomposition method for CCBs when considering credit risk. This section also shows how the complete decomposition methods take into account the interactions among value components of a CCB. Section 5 presents the analytic valuation formulae for CCBs, and validates these formulae by comparing our method with Monte Carlo simulation. Section 6 illustrates how it is intuitive and convenient to derive the formula of a specific value component of the CCBs by using our analytic formula. Besides, we further discuss assumptions on credit spread and interest rate. Section 7 presents the complete decomposition method for PCCBs and the analytic valuation formulae for PCCBs, derived by the same decomposition method. Section 8 concludes the paper.

2. Analysis on theoretical values of CCBs

2.1. General convertible description

A CCB pays a predetermined coupon and is convertible, by the holder, to a number of shares in terms of conversion price. In addition to the convertibility feature, the CCB bond may be callable by the issuer, puttable by the holder. Although a CCB may also contain other pertinent features, provisions on coupon payment, conversion, call and put are widely discussed in the literature (Anderson and Buffum, 2004; Ayache et al., 2003; Brennan and Schwartz, 1977; Ingersoll, 1977; Tsiveriotis and Fernandes, 1998). These common features of a CB are modeled in our paper as well.

For the following reasons, our paper does not get involved with all pertinent features, such as call protection[1] (Crepey and Rahal, 2011), time-varying call trigger (Ammann et al., 2003),[2] and dividend protection (Ferreira and Ouzou, 2011)[3], etc. First, the main purpose of our paper is to develop a complete decomposition method based on exotic options, which has significant advantages over those previous methods, typically including decomposition methods based on plain vanilla options, and the lattice-based numerical methods or simulation ones as well. Considering primary but not all detailed pertinent provisions in a CB would help us show more clearly and compactly the valuation framework and advantages of our method. Second, due to the limited space, in a single paper we could not discuss all pertinent features of a CB. But we would extend our studies based on this innovative pricing idea for CBs in subsequent works, where more pertinent features of CBs could be considered by referring to the papers of exotic options, such as the partial-time exotic option (Carr, 1995), window double barrier options (Guillaume, 2005), step double barrier options (Guillaume, 2010) and so on.

2.2. Notations

In this paper, we focus on the callable convertible bond (CCB). We denote its face value by B_F, conversion price by P_1, call price by B_c, the trigger price for calling by P_2, put price by B_p, the trigger price for putting by P_3, the remaining time to maturity by T, and r is the risk-free interest rate, respectively. Further, let S_τ denote the underlying stock price at any future time τ, where $0 < \tau \leq T$. Assume that there are still N payments of nominal coupons from now to maturity. Let $\tau_i (i = 1, \cdots, N)$ denote the time span from now to the ith ex-coupon date, then obviously, $\tau_N = T$. Let $C_i (i = 1, \cdots, N)$ and $R_i (i = 1, \cdots, N)$ denote the coupon amount and the coupon rate at time τ_i respectively, then $C_i = B_F R_i$.

We use the credit spread approach to consider credit risk, following the idea in Tsiveriotis and Fernandes (1998), which considers the possibility of default only when the CB issuers need to pay in cash. There are three occasions in which the CB issuer needs to pay in cash. First, prior to maturity, the issuer needs to pay coupons. Second, prior to maturity, if the issuer takes the initiative in announcing to call back the CBs, the issuer needs to pay the call price. In this case default on the payment of the call price in cash is quite impossible. Third, at maturity, the CBs will be redeemed and the issuer needs to pay $B_F + C_N$ in cash. Therefore, we consider credit risk only in the first and the third occasions by credit spreads, which increase with maturity and decrease with the underlying stock price. In this paper, credit spread is assumed to be a deterministic function of maturity and stock price. We use a discount rate, equal to the risk free interest rate plus the credit spread, to calculate present values of CBs.

Let $r_i^c (i = 1, \cdots, N)$ denote the credit spread at the payment time τ_i, $CCB(S_0, T; C, r^c)$ the theoretical value of the CCBs at current time zero

[1] Call protection was rarely included in convertible securities prior to 1982. On the issuer's right to call a CB, Call protection imposes restrictions, which may come in two forms, hard call protection (most common) and soft call protection. The former simply prohibits redemption within a prearranged period under any circumstances. The latter prohibits redemption unless the underlying common reaches a certain threshold price level (Nelken, 2000).

[2] One special case is the CBs issued by Infogrames Entertainment, which have provisions on time-varying call trigger as follows: within the period from May 30, 2000 to June 30, 2003, the call trigger is set at 250% of the early redemption price. After July 1, 2003, the call trigger is reduced to 125% of the early redemption price.

[3] The conversion price would be adjusted when an event occurs and thus change the shareholder's equity, for example, the firm issues common stock as a dividend or distribution on the common stock, the firm subdivides or combines the common stock, the firm distributes cash dividends on the common stock, and so on. When any of these events occurs, the stock price would be adjusted, usually referred to as the ex-right price. The conversion price would be adjusted accordingly. The call price is adjusted as well since it is usually proportional to the conversion price. While we could model the stock price with discrete dividend distribution, it might be too complicated to derive an analytical formula by pricing formulae of exotic options. In addition, roughly speaking, when dividends are distributed, the conversion price is adjusted accordingly such that the dividend distribution has a little impact on the value of CBs. Therefore, in our paper, we assume that the underlying stock pays no dividends or other distributions.