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Economic value of modeling covariance asymmetry for mixed-asset portfolio diversifications

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ABSTRACT

Mounting evidence from the literature points to the existence of covariance asymmetry for financial assets. That is, conditional volatility and correlation of financial returns tend to rise more after negative return shocks than after positive ones of the same size. This paper extends the literature by investigating whether investors could gain significant economic benefits from incorporating the feature into mixed-asset portfolio diversifications. We carry out the investigation for a portfolio consisting of US stock, REITs, and the risk-free asset, and find that covariance asymmetry is indeed a value-added feature for mixed-asset diversifications. This conclusion is robust to different portfolio performance metrics and asset allocation periods. More importantly, we demonstrate that the value added by modeling covariance asymmetry is unlikely to be offset by transaction costs. This leads credence to the implementability of a portfolio strategy which embeds the feature of covariance asymmetry. Our findings have important implications for fund managers and their clientele.

1. Introduction

Understanding the dynamics of volatility and correlations for financial returns is important for many financial tasks (e.g. portfolio diversification, risk management and asset pricing, etc.). This has motivated the development of a large number of econometric models and the associated empirical investigations. Interested readers may refer to Bauwens et al. (2012) for an up-to-date overview of the broad finance literature. From the large literature, one of the most salient findings suggests that both conditional volatility and correlation display asymmetric response to return shocks: they tend to rise more after negative return shocks than after positive ones of the same size (e.g. Nelson, 1991; Glosten et al., 1993; Longin and Solnik, 2001; Cappiello et al., 2006; etc.). This phenomenon is typically referred to as covariance asymmetry, due to the fact that volatility and correlation are the two constituents of covariance and both of them respond asymmetrically to financial innovations.

Given this finding, a natural question arises: what financial implications does covariance asymmetry have for fund managers and their investor clientele? In particular, could they reap tangible economic benefits by accounting for covariance asymmetry in portfolio constructions? And if so, how much the benefits would be? These questions are important. While the literature has widely explored the existence of covariance asymmetry and the econometric modeling of it, few studies have assessed the potential economic value that fund managers and their clientele could gain from incorporating the feature into portfolio decisions. Admittedly, documenting the existence of covariance asymmetry is a good first step, but such analysis *per se* is not particularly informative to investors as it falls short of answering whether there are significant economic gains from modeling covariance asymmetry. This paper takes an asset allocation perspective and aims to contribute to the literature along several dimensions. First, we will investigate whether covariance asymmetry is a value-added feature for mixed-asset portfolio diversifications. This is different than previous studies (e.g. Patton, 2004; Thorp and Milunovich, 2007) which focus on all-equity portfolios. We want to see if the feature of covariance asymmetry would bring different magnitudes of value for a mixed-asset portfolio than for just an all-equity portfolio. Second, to ensure the robustness of our findings, we will use a variety of metrics to evaluate the potential economic value added by considering covariance asymmetry. We will also discuss both the economic and statistical significance of the value. Third, we will examine the impact of transaction costs. This is a critical question: if the value added turned out insufficient to cover the higher transaction costs incurred by modeling covariance asymmetry, then it would be moot for fund managers and their clients to consider the feature. Unfortunately, this question has been consistently neglected in the relevant literature. Finally, we will explore whether our above findings are sensitive to asset allocation periodsanother missing point from the literature.

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To carry out the investigation, we consider a portfolio consisting of stocks, Real Estate Investment Trusts (REITs), and risk-free assets. We do not include bonds, because they do not display a strong feature of covariance asymmetry (e.g. Cappiello et al., 2006). REITs are included for two reasons: first, REITs, along with stocks, are rich in the feature of covariance asymmetry (e.g. Hung and Glascock, 2010; Liow, 2012; Yang et al., 2012; Zhou and Kang, 2011; etc.); second, they are a distinctive investment alternative to stocks by allowing easy access to real estate investments without directly owning or managing the underlying assets. Over the last two decades, REITs have experienced rapid market expansion and have attracted increasing attention from fund managers (Chandrashekaran, 1999). To model covariance asymmetry for this mixed-asset portfolio, we use GJR-ADCC (Glosten et al.'s (1993) GARCH–Asymmetric Dynamic Conditional Correlation of Cappiello et al., 2006). As is shown later, this multivariate GARCH model captures asymmetry in both volatility and correlation. It also accommodates all stylized facts for financial returns such as volatility clustering, and time-variations in conditional volatility and correlation. In contrast, GARCH-DCC—a nested model of GJR-ADCC (Generalized Autoregressive Conditional Heteroscedasticity–Dynamic Conditional Correlation of Engle, 2002) neglects covariance asymmetry, even though this nested model captures all other stylized facts as mentioned above. By applying both methods to a same asset allocation problem, we expect to evaluate the economic value of modeling covariance asymmetry.

We consider a risk-averse investor who forms portfolios by minimizing variance subject to a target return. We use S&P 500 Index, FTSE/NAREIT All Equity REITs Index, and the 3-month Treasury bill rates to respectively represent the three asset classes. We obtain data from January 2, 2007 through December 31, 2012. The whole sample is then divided into two periods: an estimation period (January 2, 2007 to December 31, 2010; 1000 observations) and a testing or asset allocation period (January 3, 2011 to December 31, 2012; 500 observations).We then use a recursive procedure to construct portfolios over the testing period. Overall we find that modeling covariance asymmetry yields significant economic value for mixed-asset portfolio diversifications, and the added value seems to be greater than what has been previously reported for an all-equity portfolio. More importantly, we show that the added value is unlikely to be offset by transaction costs. These results are found to hold for a different testing period (i.e. the year of 2012). This implies that covariance asymmetry is indeed an implementable value-added feature for portfolio. Our findings should benefit both fund managers and their investor clientele.

The reminder of this paper is organized as follows. Section 2 outlines the econometric methodologies. Section 3 discusses the data. Section 4 presents the empirical findings. Section 5 concludes.

2. Econometric methodologies

2.1. The asset allocation strategy

We consider an investor who allocates funds across assets by minimizing portfolio variance subject to a target return constraint. Let $\boldsymbol{\mu}$ be a vector of expected excess returns $(\mathbf{r}_{t+1} - r_f\mathbf{1})$, where $\mathbf{r}_{t+1}$ is a vector of expected returns of k risky assets, r_f is there turn of risk-free asset, and $\mathbf{1}$ is a vector of ones. Then the asset allocation problem can be written as

$$\min_{\mathbf{w}_t} \mathbf{w}_t' \boldsymbol{\Sigma}_t \mathbf{w}_t \tag{1}$$

$$s.t. \mathbf{w}_t' \boldsymbol{\mu} = \mu_p \tag{2}$$

where $\boldsymbol{\Sigma}_t \equiv E_t[(\mathbf{r}_{t+1} - \boldsymbol{\mu})(\mathbf{r}_{t+1} - \boldsymbol{\mu})']$ is the expected covariance matrix, μ_p is the target return, and $\mathbf{w}_t$ is a $k \times 1$ vector of weights on the risky assets. The solution to this optimization problem is

$$\mathbf{w}_t = \frac{\mu_p \boldsymbol{\Sigma}_t^{-1} \boldsymbol{\mu}}{\boldsymbol{\mu}' \boldsymbol{\Sigma}_t^{-1} \boldsymbol{\mu}}. \tag{3}$$

Note that we do not impose short-sales constraints so that any wealth not accounted for by $\mathbf{w}_t$ is implicitly invested in the risk-free asset, which has a weight of $(1 - \mathbf{w}_t'\mathbf{1})$.

2.2. Forecasting the conditional covariance

Implementing the above asset allocation strategy requires estimating $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}_t$. To simplify our analysis, we follow Fleming et al. (2001) by using in-sample mean return to estimate $\boldsymbol{\mu}$.Doing so allows us to focus solely on the impact of covariance structures. Another reason is that expected returns are typically estimated with far less precision than expected covariance matrices (Merton, 1980). So in what follows we mainly discuss how to estimate $\boldsymbol{\Sigma}_t$.

As a benchmark model for $\boldsymbol{\Sigma}_t$, we use the GARCH-DCC (Dynamic Conditional Correlation) model of Engle (2002). This model has been widely used. It is capable of capturing certain stylized facts of covariance structure such as volatility clustering and dynamic correlations but ignores the feature of covariance asymmetry. As an alternative that can model covariance asymmetry, we resort to a model named GJR-ADCC. On the one hand, GJR, representing the GARCH model of Glosten et al. (1993), can capture volatility asymmetry—the first layer of covariance asymmetry. A generic GJR model has the following specification:

$$h_{i,t} = \varpi_i + \alpha_i \varepsilon_{i,t-1}^2 + \gamma_i I_{t-1} \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1} \tag{4}$$

where $h_{i,t}$ is the conditional volatility for each asset of the portfolio, ε is the demeaned returns, $I_{t-1} = I(\varepsilon_{t-1} < 0)(I(\cdot)$ is an indicator function which takes on value 1 if the argument is true and 0 otherwise), and ϖ, α, γ, & β are parameters. It is easy to see that volatility asymmetry is modeled through γ, as a positive value of γ would indicate that negative return shocks generate higher volatility than positive ones of the same magnitude. γ is thus the parameter of volatility asymmetry. Setting $\gamma = 0$ reduces GJR to the standard GARCH, which neglects volatility asymmetry. On the other hand, ADCC, representing Asymmetric Dynamic Conditional Correlation model, captures correlation asymmetry—the second layer of covariance asymmetry. A generic ADDC model can be written as:

$$\boldsymbol{\Sigma}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t \tag{5}$$

where $\mathbf{D}_t$ is a $k \times k$ diagonal matrix with $\sqrt{h_{i,t}}$ on the ith diagonal, and $\mathbf{R}_t$ is the correlation matrix to be estimated. According to Cappiello et al. (2006), $\mathbf{R}_t$ can be formulated as follows:

$$\mathbf{R}_t = diag(\mathbf{Q}_t)^{-1} \mathbf{Q}_t diag(\mathbf{Q}_t)^{-1} \tag{6}$$

$$\mathbf{Q}_t = (1-a-b)\overline{\mathbf{Q}} - \phi\overline{\mathbf{N}} + a(\mathbf{z}_{t-1}\mathbf{z}'_{t-1}) + b\mathbf{Q}_{t-1} + \phi(\boldsymbol{\eta}_{t-1}\boldsymbol{\eta}'_{t-1}) \tag{7}$$

where $diag(\mathbf{Q}_t) = \left[\sqrt{q_{i,i,t}}\right]$ is a diagonal matrix containing the square root of the diagonal elements of matrix $\mathbf{Q}_t$, $\mathbf{z}_t$ is the vector of standardized residuals (i.e. $z_{i,t} = \varepsilon_{i,t}/\sqrt{h_{i,t}}$), $\overline{\mathbf{Q}} = E(\mathbf{z}_t\mathbf{z}'_t)$, $\boldsymbol{\eta}_t = I(\boldsymbol{\varepsilon}_t < 0) \circ \boldsymbol{\varepsilon}_t$ ($\circ$ denotes the Hadamard product), $\overline{\mathbf{N}} = E(\boldsymbol{\eta}_t\boldsymbol{\eta}'_t)$ and a, b, and ϕ are nonnegative scalar parameters. It is worth noting that ϕ captures