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A revealed preference test of rationing a Monte Carlo analysis

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ABSTRACT

This paper develops and evaluates a rationing linear programming procedure that uses an "efficiency index" to allow for violations of revealed preference to be attributed to "almost" optimal choices. The procedure detects rationing using U.K. data. Various Monte Carlo simulations are performed to evaluate the ability of the procedure to differentiate between violations of revealed preferences caused by random error and those caused by rationing.

1. Introduction

Rationing can severely restrict consumer choices and thereby affect estimates of consumer demand. Demand studies often account for rationing using stochastic parametric models as in Hicks (1940); Rothbarth (1941); Houthakker and Tobin (1952); Stone (1954); Stone and Rowe (1954, 1996); Neary and Roberts (1980) and Deaton (1981). In contrast, the alternative nonstochastic nonparametric rationing revealed preference procedure of Varian (1983) and Fleissig and Whitney (2011) does not require specifying a functional form and can also be used to detect the periods for which rationing on goods is binding.

There is a large literature on empirical tests of revealed preference. The approach of Afriat (1967), and developed further by Varian (1982), evaluates inequalities to determine if a set of consumption data is consistent with the axioms of revealed preference. Afriat (1967) outlines a method of testing the strong axiom of revealed preference. The procedure of Varian (1982) tests for consumer choices that are consistent with the generalized axiom of revealed preference (GARP) and allows for linear portions on indifference curves. Since these tests are nonstochastic, a single violation of GARP leads to rejecting the hypothesis of utility maximization. This problem is referred to as the "goodness of fit" and is concerned with measures of the severity of violations of GARP. It has been addressed by Varian (1990, 1991) and more recently by Echnique, Lee, and Shum (2011). Measures of the power of these tests have been proposed by Bronars (1987); Gross (1995); Adreoni and Miller (2002); Beatty and Crawford (2011), and Andreoni, Gillen and Harbaugh (2013).

In conjunction with tests of GARP, Varian (1983) also developed a test for rationing. This nonstochastic nonparametric test can detect rationing that causes violations of revealed preference. Using the information from either non-rationed goods or periods for which no rationing exists, it can be determined if virtual prices (strictly greater than observed prices) exist for the rationed goods that could make the data set as a whole (rationed and unrationed periods) consistent with utility maximization. If a feasible solution to a consumer optimization is found, it can be concluded that the data are consistent with utility maximization under rationing and a set of virtual prices for the rationed goods can be determined. Virtual prices can be used for measures of welfare loss and as a means of constructing alternative measures of inflation when rationing is present.

The modified nonparametric procedure of Fleissig and Whitney (2011) detects rationing in the U.K. during WWII. Using interwar data from Belgium, Fleissig and Whitney (2013) detect rationing in the housing market. They then use the virtual price of housing in place of the observed price to estimate a system of "free" demand equations. A two-step procedure is suggested for the nonparametric revealed preference rationing procedure. First the data are evaluated for consistency with utility maximization using Varian's (1982) GARP procedure. If GARP violations are detected during periods when none of the goods are rationed, for example during pre-war U.K., then utility maximization is rejected and no further testing is needed. If GARP violations are detected, but only in the period when goods are subject to rationing then these violations could be attributed to the effect that rationing has on utility maximization. Thus, in the second step, the LP rationing inequalities are evaluated to determine if rationing is binding on any good. If a feasible solution exists, GARP violations are attributed to binding rationing. When a feasible solution to the LP setup does not exist, then both rationing and utility maximization are rejected.

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These rationing tests are also subject to the goodness of fit problem as in GARP. A single violation in either step leads to rejecting consumer optimization. In addition, in the second step of evaluation, it is possible that a violation caused by random error can lead to the incorrect conclusion of binding rationing. That is, these tests lack the capability of filtering out violations of GARP caused by random errors from violations caused by binding rationing in the presence of random error. To address this problem, this paper develops a new procedure to evaluate utility maximization under rationing that allows for GARP violations caused by rationing and random errors. We use the Afriat Index to specify error in a consumer optimization. The properties of the new rationing test are evaluated using a Monte Carlo analysis. We find that this new test greatly reduces the probability of attributing random error associated with goodness of fit to rationing without affecting the ability to detect rationing when it is present.

2. Afriat inequalities and rationing

The revealed preference approach is frequently applied to evaluate consumer demand. Afriat (1967) and Varian (1982) develop conditions under which a well-behaved utility function rationalizes the data where $p^i = (p_1^i, \ldots, p_k^i)$ and $x^i = (x_1^i, \ldots, x_k^i)'$ are price and quantity vectors.

Afriat's Theorem. The following conditions are equivalent (Varian, 1982).

(A1) There exists a nonsatiated utility function that rationalizes the data.
(A2) The data satisfy GARP.
(A3) There exist numbers (scalars) $U^i, \lambda^i > 0$ for $i = 1,\ldots,n$ that satisfy the Afriat inequalities:

$$U^i - U^j - \lambda^j\, p^j\left(x^i - x^j\right) \le 0 \quad \text{for } i, j = 1, \ldots, n. \tag{1}$$

(A4) There exists a concave, monotonic, continuous, nonsatiated utility function that rationalizes the data.

The aim is to find a solution to a consumer optimization when choices are constrained by rationing on goods in period i:

$$\begin{aligned} &\text{Max } u(x) \\ &\text{s.t } p^i x \le m^i \\ &\quad A^i x \le b^i \end{aligned} \tag{2}$$

where A^i is a k by k diagonal matrix with each row representing rationing, but not necessarily binding constraints. Some special cases of the rationing were developed by Varian (1983) with a more general approach extended by Fleissig and Whitney (2011). The Afriat inequalities from (1) under rationing are:

$$U^i \le U^j + \lambda^j p^j\left(x^i - x^j\right) + \mu^j A^j\left(x^i - x^j\right) \tag{3}$$

with rationing on good g binding for $\mu^{gj} > 0$ and $\mu^{gj} = 0$ for non-binding constraints. Fleissig and Whitney (2011) follow Diewert (1973); Varian (1983) and Fleissig and Whitney (2003, 2005) and express rationing as a linear programming (LP) problem with a weighted objective function $\sum_{j=1}^{n}\sum_{g=1}^{h} w^g \mu^{gj}$ where w^g, the weights, are expenditures shares.

$$\begin{aligned} &\text{Min } Z = \sum_{j=1}^{n}\sum_{g=1}^{h} w^g \mu^{gj} \\ &\text{subject to} \\ &U^i \le U^j + \lambda^j p^j\left(x^i - x^j\right) + \mu^j A^j\left(x^i - x^j\right) \\ &\lambda^i > 0 \\ &\mu^{gj} \ge 0 \end{aligned} \tag{4}$$

The objective function is a weighted average (w^g average expenditure share of item g over the entire period) of the Lagrangian multipliers used to detect for rationing (μ^{gj}). If a feasible solution exists, then the data can be rationalized by a utility function for the entire period with binding rationing on goods for which $\mu^{gi} > 0$. They show that virtual prices (p^{*j}) are those prices that when substituted for observed prices of rationed goods will satisfy the Afriat inequalities (1):

$$U^i \le U^j + \lambda^j p^{*j}\left(x^i - x^j\right) \tag{5}$$

with prices of unrationed goods identical to p^j. The virtual price from Eq. (5) for the gth rationed good in period j, where A^j is a diagonal matrix with p^j on the diagonal, is:

$$p^{*gj} = p^{gj} + p^{gj}\mu^{gj}/\lambda^j = p^{gj}\left(1 + \mu^{gj}/\lambda^j\right). \tag{6}$$

Neary and Roberts (1980) show that the difference between virtual and actual prices gives a measure of the maximum amount a consumer would be willing to pay to have one extra unit of the rationed good.

3. Afriat's efficiency index and rationing tests

The LP rationing procedure rejects utility maximization under rationing for a single violation of the modified Afriat inequalities. Since there may be a margin of error in consumer optimization, Afriat (1967) and Varian (1990) suggest using an "efficiency index" to determine if violations of GARP are from sub-optimal choices. We apply the "efficiency index" to develop a procedure to test for rationing under sub-optimal choices.

To analyze sub-optimal choices that would satisfy GARP, Varian (1990, 1991) extends Afriat's (1967) approach and defines bundle x as directly revealed preferred to x^j with efficiency e, if and only if $epx \ge px^j$. GARP is then modified as:

$$\text{GARP}(e): \ \text{If } x^i R(e) x^s \text{ then } ep^s x^s < p^s x^i$$

where $0 \le e \le 1$ and R(e) is the transitive closure. The smaller "e" is, the more substantial is the difference in expenditures needed to conclude that one choice is directly revealed preferred to another. If $e = 1$ then there are no violations of GARP. When $e < 1$ the data violate GARP but the transitive closure is imposed by eliminating some of the directly revealed preferred relationships, presumably those attributed to random error. The aim is to find the largest value for e that satisfies GARP(e). It is typically assumed, as in Varian (1991), that $e \ge .95$ would indicate that violations are due to either indifference between very similar choices or slightly inaccurate measures of the difference in total expenditures between choices. Thus if $e \ge .95$, the researcher concludes that choices are rational. Varian (1990) and Gross (1995) show that the Afriat index will find data consistent with GARP if expenditure differences are relatively small even with very different consumption bundles since it will "forgive" violations when total expenditures are similar. For rationing, bundle x^i is directly revealed preferred to x^j if and only if $ep^ix^i \ge p^ix^j$ and for bundles for which some goods are rationed, $ep^{*i}x^i \ge p^{*i}x^j$, where p^* contains virtual prices for rationed goods.

The following Afriat Index LP can be used to evaluate utility maximization under rationing when the data are measured with error where the researcher selects the value for e.