Economic Modelling 42 (2014) 220–229

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

Complex dynamics of monopolies with gradient adjustment☆

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ARTICLE INFO

Article history:
Accepted 23 June 2014
Available online xxxx

Keywords:
Gradient dynamics
Boundedly rational monopoly
Continuous and discrete dynamics
Time delay

ABSTRACT

This paper aims to show that delay matters in continuous- and discrete-time framework. It constructs a simple dynamic model of a boundedly rational monopoly. First the existence of the unique equilibrium state is proved under general price and cost function forms. Conditions are derived for its local asymptotical stability with both continuous and discrete time scales. The global dynamic behavior of the systems is then numerically examined, demonstrating that the continuous system is globally asymptotically stable without delay and in the presence of delay if the delay is sufficiently small. Then stability of the continuous system is lost via Hopf bifurcation. In the discrete case without delay, the steady state is locally asymptotically stable if the speed of adjustment is small enough, then stability is lost via period-doubling bifurcation. If the delay is one or two steps, then stability loss occurs via Neimark–Sacker bifurcation.

1. Introduction

It is well known that economic agents are not fully rational, and usually prefer to employ simple rules which were previously tested (Kahneman et al., 1986). Many different output adjustment schemes have been developed. Bischi et al. (2010) offer a collection of the most important schemes in the case of oligopolies. In earlier studies linear models were examined, where local asymptotical stability implies global asymptotical stability (Okuguchi and Szidarovszky, 1999). In the last two decades an increasing attention has been given to examine the asymptotical property of nonlinear economic systems including monopolies and duopolies (Bischi et al., 2010). It is also well known that economic dynamic systems inherently incorporate delays in their actions and delay is one of the essentials for economic dynamics. Nevertheless, little attention has been given to studies on delay dynamics of boundedly rational economic agents. The main purpose of this paper is to show how delay affects dynamics in continuous- as well as in discrete-time framework.

In this paper dynamics of boundedly rational monopolies are discussed with the most popular adjustment rule in which the firm adjusts output in proportion to its marginal profit. Such an adjustment scheme is known as gradient adjustment. In contrary to best response dynamics, only local information is needed for the adjustment process which is always available to the firm. Baumol and Quandt (1964) investigated cost free monopolies in both discrete and continuous time scales and the dependence of the profit on varying price. They developed a simple adjustment mechanism that converges to the profit maximizing output. Puu (1995) has revisited this model with discrete time setting and a cubic demand function. It is shown that complex dynamics can emerge if the price function has a reflection point. Naimzade and Ricchiuti (2008) reconsidered Puu's model with linear cost and cubic price function and exhibited the birth of chaos through the period-doubling bifurcation even if the price function does not have a reflection point. Their model was then generalized by Askar (2013) with more general price functions. In both studies local asymptotical stability was analytically examined and global dynamics by computer simulations.

We consider monopoly dynamics from three different points of view. First, we are concerned with gradient dynamics in continuous-time framework while most of recent studies considered discrete-time dynamics. Second, we will further generalize the model of Askar (2013) by introducing a more general class of cost functions and determine the non-negativity condition that prevents time-trajectories from being negative. Third, we devote a little more space to exhibit that "delay" discrete-time monopoly gives rise to complex dynamics via Neimark–Sacker bifurcation while "non-delay" monopoly goes to chaos through a period-doubling cascade. This paper is strongly related to Matsumoto and Szidarovszky (2012) and Matsumoto et al. (2013). The former shows that a continuous time monopoly with two fixed time delays has a bifurcation process in which there is a period-doubling cascade to chaos and a period-halving cascade to the steady state if the length of one delay is significantly different from the length of the other delay.

☆ The authors would like to thank an anonymous referee for very helpful suggestions. They highly appreciate the financial supports from the MEXT-Supported Program for the Strategic Research Foundation at Private Universities 2013–2017, the Japan Society for the Promotion of Science (Grant-in-Aid for Scientific Research (C) 24530201 and 25380238) and Chuo University (Grant for Special Research). The usual disclaimers apply.
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http://dx.doi.org/10.1016/j.econmod.2014.06.013
0264-9993/

The latter replaces the fixed delays with continuously distributed delays in which the time window of past data is bounded from below and its length is fixed.

The paper is organized as follows. In Section 2, the model is presented and the existence of the unique profit maximizing output level is proved. In Section 3, the dynamic system with the gradient adjustment is constructed in continuous time scale. Without delayed information the continuous model is always locally asymptotically stable, and the stability can be lost if only delayed revenue information is available. In Section 4, the continuous-time model is discretized. It is analytically shown and numerically confirmed that the discrete-time model can give rise to aperiodic oscillations through a period-doubling bifurcation or Neimark–Sacker bifurcation whether the model involves "delay" or not. In the final section, concluding remarks are given.

2. Continuous model

A general inverse demand function as well as a general cost function are considered in a monopoly. Let $p(q) = a - bq^{\alpha}$ be the price function and $C(q) = cq^{\beta}$ be the cost function. The case of $\alpha = 3$ and $\beta = 1$ was examined by Naimzade and Ricchiuti (2008), the more general case with any $\alpha \geq 3$ and $\beta = 1$ was discussed by Askar (2013). We move one step forward from their studies and consider the case where both α and β are greater than 1.

Assumption 1. $\alpha > 1$ and $\beta > 2$.

The profit of the monopoly is given as

$$\pi(q) = (a - bq^{\alpha})q - cq^{\beta}. \tag{1}$$

Notice that

$$\pi'(q) = a - b(\alpha+1)q^{\alpha} - c\beta q^{\beta-1} \tag{2}$$

and

$$\pi''(q) = -b\alpha(\alpha+1)q^{\alpha-1} - c\beta(\beta-1)q^{\beta-2} < 0, \tag{3}$$

so $\pi(q)$ is strictly concave in q, furthermore

$$\pi(0) = 0, \ \lim_{q\to\infty} \pi(q) = -\infty \ \text{ and } \ \pi'(0) = a > 0.$$

Therefore there is a unique profit maximizing output $\overline{q}$ which is the unique solution of equation

$$b(\alpha+1)q^{\alpha} + c\beta q^{\beta-1} = a. \tag{4}$$

The left hand side is zero at $q = 0$, converges to ∞ as q tends to infinity and is strictly increasing. So the value of $\overline{q}$ can be obtained by using simple numerical methods (see, for example, Szidarovszky and Yakowitz, 1978). The remaining part of this section has two subsections. In Section 2.1, the condition of local asymptotical stability is derived. In Section 2.2, we examine the effects caused by changing values of α and β on stability.

2.1. Stability

Assuming gradient dynamics, the firm adjusts its output according to the following differential equation:

$$\dot{q}(t) = k\pi'(q(t))$$

or

$$\dot{q}(t) = k\left(a - b(\alpha+1)q(t)^{\alpha} - c\beta q(t)^{\beta-1}\right). \tag{5}$$

This is a nonlinear system with positive adjustment coefficient k. The unique steady state of this system is the profit maximizing output $\overline{q}$. Local asymptotic stability can be examined by linearization. The linearized equation can be written as

$$\dot{q}_{\varepsilon}(t) = k\pi'(\overline{q})q_{\varepsilon}(t)$$

or

$$\dot{q}_{\varepsilon}(t) = k\left(-b\alpha(\alpha+1)\overline{q}^{\alpha-1} - c\beta(\beta-1)\overline{q}^{\beta-2}\right)q_{\varepsilon}(t) \tag{6}$$

where $q_{\varepsilon}(t) = q(t) - \overline{q}$. Since the multiplier of $q_{\varepsilon}(t)$ on the right hand side is negative, the steady state $\overline{q}$ is locally asymptotically stable. Let $r(q)$ denote the right hand side of Eq. (5). Notice that it strictly decreases in q and $r(\overline{q}) = 0$. So $r(q) < 0$ if $q > \overline{q}$ and $r(q) > 0$ if $q < \overline{q}$. Therefore if $q(0) < \overline{q}$, then $q(t)$ strictly increases and converges to $\overline{q}$; if $q(0) > \overline{q}$, then $q(t)$ strictly decreases and converges to $\overline{q}$; and if $q(0) = \overline{q}$, then $q(t) = \overline{q}$ for all $t \geq 0$. Hence system (5) is global asymptotically stable, and with $q(0) > 0$, the entire trajectory $q(t)$ remains positive. Although this result is well known, we formally state it as it is a benchmark of this study:

Theorem 1. *For any k, the non-delay continuous time model (5) is locally and globally asymptotically stable.*

Assume next that the monopoly receives marginal revenue information with a positive delay $\tau > 0$, which could be due to delay price information. Then system (5) becomes a delay differential equation,

$$\dot{q}(t) = k\left(a - b(\alpha+1)q(t-\tau)^{\alpha} - c\beta q(t)^{\beta-1}\right). \tag{7}$$

The derivatives of the right hand side with respect to $q(t - \tau)$ and $q(t)$ are

$$\frac{\partial \dot{q}(t)}{\partial q(t-\tau)} = -kb\alpha(\alpha+1)q(t-\tau)^{\alpha-1}$$

and

$$\frac{\partial \dot{q}(t)}{\partial q(t)} = -kc\beta(\beta-1)q(t)^{\beta-2},$$

so the linearized equation has the form

$$\dot{q}_{\varepsilon}(t) = -kAq_{\varepsilon}(t-\tau) - kBq_{\varepsilon}(t) \tag{8}$$

with

$$A = b\alpha(\alpha+1)\overline{q}^{\alpha-1} \ \text{ and } \ B = c\beta(\beta-1)\overline{q}^{\beta-2}.$$

We know that the system is locally and globally asymptotically stable for $\tau = 0$. In order to find stability switches with increasing value of τ, assume that $q_{\varepsilon}(t) = e^{\lambda t}u$, and substitute it into Eq. (8) to obtain:

$$\lambda e^{\lambda t} = -kAe^{\lambda(t-\tau)} - kBe^{\lambda t} \tag{9}$$

so the characteristic equation is a mixed polynomial–exponential equation

$$\lambda + kB + kAe^{-\lambda\tau} = 0. \tag{10}$$

With any stability switch, $\lambda = i\omega$ with $\omega > 0$, so

$$i\omega + kA(\cos\omega\tau - i\sin\omega\tau) + kB = 0 \tag{11}$$