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A semiparametric approach to value-at-risk, expected shortfall and optimum asset allocation in stock–bond portfolios☆

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ABSTRACT

This paper investigates stock–bond portfolios' tail risks such as value-at-risk (VaR) and expected shortfall (ES), and the way in which these measures have been affected by the global financial crisis. The semiparametric *t*-copulas adequately model stock–bond returns joint distributions of G7 countries and Australia. Empirical results show that the (negative) weak stock–bond returns dependence has increased significantly for seven countries after the crisis, except for Italy. However, both VaR and ES have increased for all eight countries. Before the crisis, the minimum portfolio VaR and ES were achieved at an interior solution only for the US, the UK, Australia, Canada and Italy. After the crisis, the corner solution was found for all eight countries. Evidence of "flight to quality" and "safety first" investor behaviour was strong, after the global financial crisis. The semiparametric *t*-copula adequately forecasts the outer-sample VaR. These findings have implications for global financial regulators and the Basel Committee, whose central focus is currently on increasing the capital requirements as a consequence of the recent global financial crisis.

1. Introduction

Stocks and bonds are two of the most frequently considered asset classes in portfolio asset allocation strategies. Recently, several studies have found that the stock–bond returns correlation is negative, and it becomes more negative during crises. This phenomenon has important implications for portfolio asset allocation decisions for investors, such as portfolio diversification, risk hedging, dynamic asset allocation and portfolio rebalancing. The recent global financial crisis (GFC) had a profound impact on many developed countries. In view of the expected strong negative correlation between stock and bond returns during the GFC, and the fact that value-at-risk (VaR) and the expected shortfall (ES) are popular measures of the downside market risk, one of the aims of this paper is to investigate the relationship between these two risk measures of stock–bond portfolios and the GFC.

The main contribution of this paper is the modelling of the joint distributions of stock–bond returns by employing semiparametric copula models, which fully capture the dependence structure and the distinct characteristics of bond and stock returns. The potential benefits of our investigation come from the use of flexible copula models for estimating the portfolio tail risks such as VaRs and ES of Australia and the G7 countries. Furthermore, in the existing literature, fully parametric methods such as multivariate normal or t-distributions are employed for estimating downside tail risks, which form the capital requirements for risk management purposes. After the GFC, the financial community has lost faith in such parametric statistical models, because these models do not capture the underlying tail dependence among assets' returns, and the large losses during crises are inadequately modelled. As a result, these widely used models vastly underestimated the tail risks during the GFC, which resulted in inefficient risk management. This paper, on the other hand, uses semiparametric copula models which are flexible for capturing the underlying tail dependence as well as adequately modelling large tail losses. Thus, the tail risks can be reliably estimated, and we believe that the financial community will have some faith in such flexible models. To the best of our knowledge, this line of research has not previously been pursued in the empirical finance literature. The reason for including the G7 countries is that the GFC has had catastrophic effects on these major economies. Australia is included because it is widely known that the Australian financial market and its economy have been somewhat resilient to the GFC.

Moreover, this paper differs from previous studies in methodology in that a pattern recognition device known as the chi-plot[1] is used in this paper. This plot can reveal the significance of dependence that is closely associated with copulas. See, for example, Boero et al. (2010) for an application of the chi-plot which is explained briefly in

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[1] Introduced first by Fisher and Switzer (1985).

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Section 5 of this paper. The semiparametric copula models[2] are used to measure the inter-relationships of stock–bond markets, while previous studies have mostly used fully parametric copula models in a similar context; see, for example, Patton (2006). The semiparametric procedure, which is found to be robust to mis-specification of marginal distributions (Kim et al., 2007a), is employed for estimating the copula models which, in turn, are used for estimating risk measures such as VaR and ES in our study. Following Genest et al. (2009), two rank-based goodness-of-fit testing procedures, known as "blanket tests", are employed to assess the suitability of some copulas and select the one that fits the data well.

The existing literature on the stock–bond relationship is broadly in agreement as to the way in which the stock–bond dependence varies due to regime changes. Early studies used the simple linear correlation[3] to measure the degree of the stock–bond dependence and have established empirical evidence of varying correlation over long horizons; see, for example, Fleming et al. (1998) and Connolly et al. (2005). For a given asset allocation, these authors have found that the stock–bond portfolio diversification is also changing.[4] Motivated by the work of Caballero and Krishnamurthy (2008) on "collective risk management in a flight to quality episode", the empirical investigation on the flight to quality phenomenon in stock and bond markets received considerable attention in the recent literature. See, for example, Dirk et al. (2009), Marie et al. (2012), and among others.

The copula function has become very popular for modelling various dependence structures in multivariate data arising in areas such as economics, finance and insurance; see Cherubini et al. (2011) for examples of applications of copulas in finance. One does not need to assume any parametric form for the marginals, as it has no role in the specification of copula.[5] The joint *cdf*, estimated via a copula approach, contains all the information about the marginals as well as all the information about the dependence structure that is captured by the copula. In addition, copula functions are able to capture different types of asymmetric dependence and the rich patterns of tail behaviour of joint distribution. Because of these properties, the copula models are found to be appropriate for capturing the underlying true dependence in financial time series.

The semiparametric copula modelling of joint distributions would be very attractive due to its flexibility and robustness. In this approach, a nonparametric method is used to estimate the univariate margins, the shapes of which are largely unknown in practice, and then a parametric copula is fitted to the joint distribution. Thus, the estimates of tail risks of portfolios by semiparametric copula models are expected to be more reliable than their parametric counterparts. However, Artzner et al. (1997, 1999), showed that ES is a coherent risk measure, while VaR is not. Semiparametric copulas for computing the VaR and ES would help internationally active banks and other financial institutions to estimate reliable VaR and ES, and hence the right level of capital requirements which is currently the central focus of global regulators and the Basel Committee. Furthermore, the copula has become a popular and more advanced alternative to linear correlation in the area of financial risk management where extreme events need to be captured for accurate estimation of VaR and ES. See the survey paper by Embrechts et al. (2003) in regard to modelling extreme events.

Assuming independence between the stocks and bonds, Hyung and de Vries (2007) arrived at an interior solution, which tends to select the asset with the thinnest tail.[6] By adopting the semiparametric copula approach, we model the underlying dependence between the bond and stock returns which will, in turn, allow one to compute the VaR close to the true value and the bond weight that provides the minimum VaR and ES. Using this approach, the interior solution was found for five countries (the US, the UK, Australia, Canada and Italy) before the GFC. However, the corner solution was found for all eight countries after the GFC. That is, a 100% bond investment provided the minimum VaR and ES.

The rest of the paper is planned as follows: the next section describes the data and the results of some preliminary analysis. Section 3 describes the methodological aspects of copula models. Section 4 presents the estimation methods for copula models, as well as the VaR and ES computations via the semiparametric copula approach. Section 5 describes the methodology of chi-plot, and Section 6 reports and analyses the empirical results. Finally, Section 7 concludes the paper. In Appendix A, we briefly discuss a few copula functions with various asymmetric and tail properties, which have been found to capture atypical features of financial data.

2. Data and some preliminary analysis

The government bond index was collected for Australia and the G7 countries from the Thomson Reuters Datastream for the period 01/07/2003 to 17/08/2011. The stock price indices include: (1) Australian All Ordinaries Index [AUS]; (2) TSX Composite Index of Canada [CAD]; (3) CAC 40 Index of France [FRA]; (4) DAX 30 Performance Index of Germany [GER]; (5) FTSE MIB Index of Italy [ITL]; (6) NIKKEI 225 Index of Japan [JPN]; (7) FTSE 100 Index of the United Kingdom [UK]; and (8) Dow Jones Industrials Index of the United States [US]. The sample period includes the period of the global financial crisis (GFC) triggered by the sub-prime mortgage crisis in mid-2007. The starting date of our sample period was chosen to be mid-2003 to avoid confounding with the effect of the IT bubble burst in 2001. Since one of our aims is to examine the impact of the GFC on the measures such as stock–bond dependence, the VaR and ES of the stock–bond portfolio as well as on the optimum asset allocation, we split the sample period into two: the pre- and the post-GFC periods 01/07/2003–13/07/2007, and 16/07/2007–17/08/2011, respectively. The return, main variable of interest, is defined as $X = log(P_t) - log(P_{t-1})$, where P_t is the price index at time t.

2.1. Filtering the returns

Let X_{1t} and X_{2t} denote the bond returns and stock returns, respectively, at time t ($t = 1, \ldots, T$). For each country and sample period, the best fitting univariate GARCH type model of the following form was estimated:

$$X_{jt} = \gamma_{j0} + \sum_{k=1}^{m_j} \gamma_{jk} X_{j,t-j} + R_{jt}. \quad R_{jt} = h_{jt}\varepsilon_{jt}, \quad h_{jt}^2 = \alpha_{j0} + \sum_{k=1}^{m_j} \alpha_{jk}\varepsilon_{j,t-k}^2,$$

where $(\varepsilon_{1t}, \varepsilon_{2t})$ are assumed to be independent and identically distributed ($t = 1, \ldots, T$). The filtered returns and standardized residuals

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[2] Contributors to semiparametric methods include Genest et al. (1995), Tsukahara (2005), Chen and Fan (2006), Kim et al. (2007a,b) and Kim et al. (2008).

[3] See the studies by Ilmanen (2003), Jones and Wilson (2004) and Li (2004) on modelling the stock–bond dependence using various methodologies.

[4] Andersson et al. (2008) examined the impact of inflation and economic growth expectations and the perceived stock market uncertainty on the time-varying correlation between stock and bond returns. Their results indicated that stock and bond prices move in the same direction during periods of high inflation expectations, while a negative stock–bond return correlation seemed to coincide with the lowest levels of inflation expectations.

[5] Sklar (1959) proved that a multivariate joint cumulative density function (*cdf*) could be fully and uniquely characterized by its continuous marginal functions and a copula function. The Sklar theorem allows the marginals and the copula components to be estimated separately and independently of each other. See also Rosenblatt (1952) for some remarks on multivariate transformation.

[6] See, for example, early contributors: Roy (1952), Arzac and Bawa (1977) and Jansen et al. (2000) for studies on portfolio selection, limited downside risk and safety first investors.