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Robust analysis for downside risk in portfolio management for a volatile stock market

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ABSTRACT

Variance and downside risk are different proxies of risk in portfolio management. This study tests mean–variance and downside risk frameworks in relation to portfolio management. The sample is a highly volatile market; Karachi Stock Exchange, Pakistan. Factors affecting portfolio optimization like appropriate portfolio size, portfolio sorting procedure, butterfly effect on the choice of appropriate algorithms and endogeneity problem are discussed and solutions to them are incorporated to make the study robust. Results show that downside risk framework performs better than Markowitz mean–variance framework. Moreover, this difference is significant when the asset returns are more skewed. Results suggest the use of downside risk in place of variance as a measure of risk for investment decisions.

1. Introduction

Risk-return relationship is long considered as the backbone in portfolio management (Elton et al., 2003). Firms hedge and construct portfolios to guard against financial risk as a part of risk mitigation strategy. Measures of risk used, however, are largely debated in the extant literature[3] with no consensus on the choice of risk measure. Traditionally, in a mean and variance (MV) framework, the latter has been used as a proxy for risk which, in turn, assumes that the investor gives equal weights to both upside and downside risks (Markowitz, 1952, 1959). The downside risk (DR) framework, on the other hand, is largely based on the concern of investors for safety from a disaster rate (Estrada, 2002; Post and Levy, 2005; Roy, 1952).

Motivation for testing the two frameworks in a volatile market is driven by the fact that appropriate measures of risk become crucial to individual and organizations in markets that are marked by high uncertainty. During volatile times, many investors are concerned and question their investment strategies in terms of asset allocation. Moreover, robust studies require addressing different issues[4] related to portfolio optimization for both frameworks which previous studies lacked. Though solutions are proposed in different literatures but a comprehensive study for investors, institutional investors and researchers alike for financial modeling is needed. Primarily, asset allocation ensures value for the stakeholders in financial markets. This study will help in anticipating portfolio risk for the investor, explain the market behavior, the nature of investment and can be pivotal for an institutional demand of common stocks.

Harlow (1991) and Foo and Eng (2000) compare MV and DR frameworks but they ignore the effect of skewness and high correlation between variance and downside risk. Our contribution to the existing literature is that we study the alternative models based on sorted portfolios on skewness and measures of risk; variance and downside risk which is previously neglected. Secondly, we address a number of key issues like appropriate portfolio size, sorting procedures, butterfly effect on the choice of appropriate algorithms and the endogeneity problem. These issues have not been discussed in one study and are normally considered as contributing to the contrasting results in the empirical studies.

Thirdly, we contribute to the relevant literature by providing empirical evidence on both MV and DR frameworks in a volatile emerging market such as Pakistan. Haque et al. (2004) comment that the safety-first rule offers minimization of the chance of large negative returns. This is appropriate for emerging markets as their equity distributions are subject to extreme returns. Lastly, we use an index to assess differences between MV and DR frameworks. This index will help in assessing the magnitude of difference in portfolio composition under alternative models.

The results of this study report that DR framework is more efficient than MV framework especially when skewness is high. On the other

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[3] See e.g., Markowitz (1952, 1959), Estrada (2002) and Post and Levy (2005).
[4] i.e. appropriate portfolio size, sorting procedures, butterfly effect on the choice of appropriate algorithms and the endogeneity problem.

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hand, the former is relatively less efficient compared to the latter when skewness is on the low side. These results are based on different portfolio sorting methods. It can be safely concluded that downside risk, as a proxy of risk, is a better measure compared to the variance. Moreover, it is reported that difference in portfolio composition under alternative models is also substantial that cannot be ignored.

The article is organized as follows; Section 1 is the introduction and the next section starts with a brief description of proxies of risk; variance and downside risk. This is followed by discussions on factors affecting portfolio optimization. Section 2 covers data and methodology while Section 3 presents the results. Last section is conclusion and recommendations.

1.1. Variance and downside risk as a measures of risk

Markowitz (1952) assumes that the investment decision is made on the parameters of return and risk. Stock returns are assumed jointly normal to justify the variance as a proxy for risk (Hwang and Pedersen, 2002). The criterion to be jointly normal is that the stock returns have to be individually normal as well while the converse is not true (Shanken, 1982; Zhou, 1993). Contrary to the condition of normality assumed, stock returns are found to exhibit skewness and kurtosis.[5] These findings make variance as the proxy of risk questionable, especially under large departure from normality and when the distribution is severely asymmetric (Athayde and Flôres, 2004; Chunhachinda et al., 1997; Harvey et al., 2010; Jondeau and Rockinger, 2006).

Similarly, Roy (1952) argues that the investor care for disaster following safety-first rule. Moreover, there is evidence of investors assigning different weights to upside and downside risk (Estrada, 2002, 2007; Gul, 1991; Kahneman and Tversky, 1979; Post and Levy, 2005). As investors prefer the safety from disaster and, furthermore, stock returns do not depict the normal distribution, DR measure is a better choice over variance as a proxy for risk (Atwood et al., 1988; Foo and Eng, 2000; Harlow, 1991; Sing and Ong, 2000; Swisher and Kasten, 2005).[6] Cheng and Wolverton (2001) compare MV and DR frameworks and conclude that the latter is a viable alternative compared to the former. Brogan and Stidham (2005) report that DR is consistent with the way investors perceive risk. Moreover, Schindler (2009) studies the co-movements between various asset returns and questions the application of MV optimization. Kroencke and Schindler (2010), Kuzmina (2011) and Sévi (2013) state the practical viability of DR framework in portfolio optimization.

1.2. Optimization model

Insight to the comparative analysis of the two alternative risk measures; variance and downside risk, leads to the construction of efficient frontiers using convex optimization. Markowitz (1952, 1959) primarily asserts that variance is the only measure of risk. He also contests that all, apart from systematic risk, can be diversified away. In this case, the variance of a portfolio is weighted covariance between individual securities as:

$$\sigma^2_{MV-P} = \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} W_i W_j \sigma_{ij}. \tag{1}$$

Subject to the following constraints:

$$C1: \sum_{i=1}^{n} W_i = 1$$
$$C2: W_i > 1 \text{ for short sales not allowed}$$

where σ^2_{MV-P} is the variance of the portfolio based on MV framework, W_i and W_j are weights of individual securities i and j and σ_{ij} is the covariance between securities i and j. $C1$ implies that all weights should be equal to 1. $C2$ is short sales not allowed indicating all weights are non-negative.

Using Eq. (1), Markowitz's (1952) efficient frontier can be constructed assuming efficient diversification and without unlimited borrowing and lending following Harlow (1991), Foo and Eng (2000), Boasson et al. (2011) and Rasiah (2012). DR framework is incorporated in Eq. (1) by replacing variance by proxy for downside risk as Asymmetric Lower Partial Moments (ALPM)[7] subject to the same constraints as in Eq. (1) as follows:

$$\sigma^2_{DR-P} = \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} W_i W_j ALPM_{ij} \tag{2}$$

where σ^2_{DR-P} is the variance of the portfolio based on DR framework and $ALPM_{ij}$ is the covariance between securities i and j such that the covariance between securities i and j is not necessarily equal to j and i.[8] Researchers like Harlow (1991), Foo and Eng (2000), Boasson et al. (2011) and Rasiah (2012) construct efficient frontiers for both the alternative models using Eqs. (1) and (2). Convex optimization is used and comparison between variance and downside risk as a measure of risk is investigated. However, there are factors that cannot be ignored that influence the optimization process, and in return, affecting the efficient frontiers. They are discussed in the following section.

1.3. Factors affecting portfolio optimization

Portfolio optimization is the process which identifies the appropriate proportions of various assets to be held in a portfolio. The criterion for it is based on portfolio returns and the dispersion of returns along with the covariances involved in the portfolio optimization process. Different factors like appropriate portfolio size, sorting procedures, butterfly effect on the choice of appropriate algorithms and the endogeneity problem affect this process. These factors are important to make the process as well as the study robust. This study discusses these factors comprehensively and applies them in MV and DR frameworks to yield robust results.

1.3.1. Portfolio size determination

Inappropriate allocations, the inclusion of inappropriate assets and non-uniqueness of the optimizer solutions are common problems in portfolio optimization arising due to the absence of an appropriate portfolio size. The nexus of feasible and appropriate portfolio size is directly linked to the benefits of diversification. Evans and Archer (1968) propose that around 10 stocks are benchmarked to attain the benefits of diversification. Similarly, Elton and Gruber (1977) use equal-weighted portfolios and report that 10–15 stocks in a portfolio is an appropriate figure. Statman (1987) concludes that 30–40 stocks are sufficient for a well-diversified portfolio. Fama and French (1992) use 25-stock portfolio for their study while Byrne and Lee (2000) advocate the use of 20–40 stocks in a portfolio for naïve investors to make a well diversifiable portfolio.

[5] Evidence for skewness see Eftekhari and Satchell (1996); Bekaert and Harvey (1997), Hwang and Pedersen (2002), Dufour et al. (2003), Sheikh and Qiao (2010) and Ramos et al. (2011).

[6] For a comprehensive literature review on both risk measures, see Abbas et al. (2011).

[7] Bawa (1975), Fishburn (1977) and Bawa and Lindenberg (1977) are the major three who contributed in specifying the proxy for downside risk.

[8] Unlike Markowitz (1952) where covariance of security i and j is equal to covariance of j and i.