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Modeling loss given default with stochastic collateral

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ABSTRACT

This article addresses to the appropriate modeling of loss given default (LGD) for the retail business sector. We assume small or mid-size loans that are assigned in a standardized way and collateralized by residential or commercial property. The focus on this specific type of loans entails two major advantages: Firstly, reduction of complexity is followed by easier-to-grasp methodology and increased handiness of results when comparing with other recent approaches in the field. Secondly, the focussing allows to take into account the characteristic properties of the housing market and its underlying uncertainty and so choose a tailor-made modeling for the collateral. The choice of an exponential Ornstein–Uhlenbeck diffusion as the stochastic process of the collateral combines the desirable features with the charm of analytical solvability which seems to be of advantage as regards acceptance among practitioners. Further key improvements of this approach are the explicit consideration of loan ranking, the disentanglement of the time of default and the time of liquidation as well as the introduction of liquidation cost.

1. Introduction and motivation

Loss given default (LGD) is one of the key measures when modeling and managing credit risk. It captures the percental loss the bank faces in case of a defaulting obligor. Since 2006, the Basle Committee on Banking Supervision (BCBS) allows banks to use their own rating approaches for the purpose of calculating the required equity for credit collateralization — i.e. a so-called *Internal Rating Based Approach* (IRBA). This concept stipulates the idea of expected loss as a product of three factors:

$$E[L] = PD \cdot EAD \cdot LGD.$$

The first factor *PD* represents the *probability of default* (PD), the second one *EAD* denotes the amount of unredeemed outstanding debt at the moment the obligor defaults, the *exposure at default* (EAD). The third component yet is the *loss given default* (LGD) as percentage of nonrecoverable debt related to EAD. Banks using the advanced rating approach are allowed to estimate these parameters single-handedly by means of internally developed methods. If one assumes that the EAD component is predictable to a great extent by means of amortization schedules, the problem reduces to an accurate estimation of LGD and PD. For the bank, reliable estimates of each component are important in equal measure: Correctly estimated (and low) values for LGD and/or PD lead to lower expected loss and therefore to lower capital requirements and a reduction of risk capital.

Before proceeding it is worth concretizing the concept of expected loss as introduced in the equation above. Let us consider a portfolio with N debtors. For each of them i, $i = 1, \ldots, N$, let D_i be the digital random variable that indicates whether the very debtor defaults with possible realizations for D_i to be 1 in case of default and 0 if no default occurs, i.e. D_i is a digital random variable or indicator function. Consequently, the random loss L_i in absolute terms is

$$L_i = D_i \cdot LGD_i \cdot EAD_i,$$

where LGD_i and EAD_i are the percentage loss and the outstanding debt in case of default of the ith obligor. Taking expectations we receive

$$\begin{aligned} E[L_i] &= E[D_i \cdot LGD_i \cdot EAD_i] \\ &= E[D_i \cdot E[LGD_i \cdot EAD_i | D_i]] \\ &= P(D_i = 1) \cdot E[LGD_i \cdot EAD_i | D_i = 1] + P(D_i = 0) \cdot E[LGD_i \cdot EAD_i | D_i = 0] \\ &= P(D_i = 1) \cdot E[LGD_i \cdot EAD_i | D_i = 1], \end{aligned}$$

where for the last line we identified the case of no default with the absence of any loss. Slightly simplifying notation by PD_i for $P(D_i = 1)$ and assuming deterministic exposure at default EAD_i, we obtain

$$E[L_i] = PD_i \cdot E[LGD_i | D_i = 1] \cdot EAD_i.$$

When we compare this to the equation provided by the BCBS, we state that the notation used of the latter implicitly assumes that both the percentage loss and the exposure at default are known with certainty. Deviating from that we assume in this article uncertainty with respect to the

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LGD and identify the LGD in the BCBS sense with the expected loss given default $E[LGD_i|D_i = 1]$.

Even though the equations and the discussion above stress the importance of LGD for banks, the scientific debate seems to be biased towards a very intense discussion about default probabilities leading with highly ambitious approaches (for a recent example using Extreme Value Theory, see Tsai and Chen (2011)) On the other hand, sophisticated models for LGD or recovery rates are rather scarce. One reason for that may be the fact that the conceptual requirements of the BCBS with respect to the loss given default had not been specified that clearly until recently. Another reason could be that in a general framework the modeling of LGD cannot be reasonably done without simultaneously modeling the PD component. There are a number of empirical articles that indicate a dependence of PD and LGD: Altman et al. (2001), Altman et al. (2005), Caselli et al. (2008) and Acharya et al. (2007) use regression based models to show that the economic cycle of an industrial sector or of the whole economy explains both probabilities of defaults and historically realized values of loss given default in a significant way. Hu and Perraudin (2006) use Extreme Value Theory to prove a direct correlation of PD and LGD in the US bond market. Bade et al. (2011) investigate corporate loans and identify a correlation between the default process (i.e. PD) and the process of recovery values (i.e. LGD) by means of maximum likelihood methods.

The existing literature concerning theoretical aspects of LGD, however, restricts itself to a very general and hardly applicable view of the topic. The theoretical models usually account for the possible dependence of PD and LGD in one of the following ways: Frye (2000), Dev and Pykhtin (2002), Hillebrand (2005), Van Damme (2011) and Jacobs (2011) model the recovery rate as one random variable and the assets of the obligor as a second one and let both of them be driven by one latent factor. Jokivuolle and Peura (2003) and Pykhtin (2003) choose correlated stochastic processes for the firm value on the one hand and the value of the collateral on the other hand. More recently, Schäfer and Koivusalo (2013) assume that the market value of the defaultable firm is driven by two correlated diffusions representing market risk and idiosyncratic risk, respectively. In this framework, the authors are able to derive an explicit functional relationship between PD and LGD. Throughout all these approaches, the resulting formulas are highly complex but still vague for lack of concretion towards a realistic and practice-oriented type of collateral. Consequently, trying to catch 'all by one', these approaches end up at the lowest common denominator. Typically, this common denominator is found to be geometric Brownian motion which then again can neither satisfy researchers nor practitioners.

As we acknowledge the impossibility to capture the heterogeneity of different LGD estimation problems within one general and still powerful model, we enter the alternative path of specification: In this paper we focus on one single but typically quite important portion of a bank's credit portfolio, the part of the retail business where loans are collateralized by residential or commercial property. We look at loans that are conferred in standard way to an obligor, which typically is represented by a private individual or a small or mid-size company.[3]

We now explain how this focus allows us to neglect the phenomenon of correlated PD and LGD by means of economic latent factors described above and thereby jettison part of the methodological over-complexity. This modeling is supported by some recent empirical results of Grunert and Weber (2009) and Grunert (2010): Both articles use default histories of small bank loans to conclude that there is no significant correlation between a bunch of economic indicators and the realized recovery rates. On the other hand, the assumption of independence for the retail business is not at odds with the other studies cited above that confirmed a general correlation of default probability and loss given default: The relationships detected in Altman et al. (2001), Altman et al. (2005), Hu and Perraudin (2006) as well as Bade et al. (2011) relate to defaults of publicly traded bonds. Acharya et al. (2007) derive their insights from some data covering large-cap credit portfolios. Evidently, both types of financing are crucially different to the case of a classical mortgage-backed loan. Only the work of Caselli et al. (2008) addresses their analysis to small and mid-size bank loans of an Italian bank. The authors again receive some general evidence for a correlation between LGD and macroeconomic factors, yet they explicitly stress that this evidence disappears when choosing only the loans that are collateralized by residential property.

Summarizing, for the special case of a mortgage-backed private loan, the assumption of independence of the local housing market on the one hand and the solvency of the single obligor on the other hand, seems to be feasible or at least not a major limitation. But what we earn is much more: We obtain an increased analytic manageability which should also increase acceptance among practitioners considerably. Meanwhile, the model reduction fans out a multitude of possibilities for an adequate modeling of the collateralizing asset. As we focus on residential property, we have a look on mathematical models of real estate markets (commercial and residential).

With respect to the descriptive and empirical level, there is early work of Case and Shiller (1989), Case and Shiller (1990) and Hosios and Pesando (1991). All three emphasize the incompleteness of real estate markets and elaborate the phenomenon of serial correlation to be a key ingredient of an appropriate mathematical model. Furthermore, seasonality seems to play an important role for the studies dealing with local housing prices in Chicago (Case/Shiller) and Toronto (Hosios/Pesando). Englund and Ioannides (1997) affirm these relationships when investigating international data sets. A number of articles try to ascribe these stylized facts to search-theoretic (see Wheaton (1990), Krainer (2001), Piazzesi and Schneider (2009), Novy-Marx (2009) and Díaz and Jerez (2013)) and/or behavioristic (see e.g. Hott (2011)) mechanisms.

Despite these insights the early literature dealing with derivative pricing in the real estate sector is based on the assumption of complete markets and geometric Brownian motion as stochastic model (see Titman and Torous (1989), Buttimer et al. (1997) and Björk and Clapham (2002)). Later models keep geometric Brownian motion as driving diffusive process but introduce equilibrium models to account for market incompleteness (see Geltner and Fisher (2007) and Cao and Wei (2010)). Crawford and Fratantoni (2003) suggest ARIMA- and GARCH-models for a realistic mapping of house price indices. The recent work of Fabozzi et al. (2010) again stresses the need of a mathematical model that incorporates serial correlation and provides new empirical evidence. The model the authors suggest ties in with the approaches of Lo and Wang (1995) and Jokivuolle (1998) who deal with other serially correlated assets. Finally, Fabozzi et al. (2012) point out that the property of serial correlation must be regarded as a central requirement when modeling any kind of real estate. The process these authors use and which also Perelló et al. (2008) use in a slightly different context with stochastic volatility, is called exponential Ornstein–Uhlenbeck process. Against the background described above, we also adopt this stochastic process for the purpose of modeling the value of the collateralizing residential property.

The basic idea of this article is to use a conceptual analogy of option pricing theory for LGD modeling. More precisely, we interpret the loss profile of a debtor at default as kind of a put option, where the underlying is identified by the vale of the collateral. Our model for LGD estimation shows a number of advantages with respect to practical use that to the best of our knowledge have not been worked out within the literature before: Firstly, we explicitly differentiate between the time of default and the time of liquidation. This separation makes allowance for the fact that the liquidation procedure is preceded by several steps of administrative and/or legal character, which leads to a delay between the time of default and the start of the liquidation (see also Gürtler and Hibbeln (2013)). Secondly, we introduce a cost factor that captures the liquidation efforts that may also affect the amount of loss. This approach acknowledges the requirement of the Basle committee, which provides workout costs to be included in the definition of LGD. Thirdly, our model easily captures the existence of loan-specific rank structures.

[3] Large-size engagements with international companies are excluded, as they are crucially different for being structured in a much more complex way.