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An operational, nonlinear input–output system ☆

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ABSTRACT

We develop a scale-dependent nonlinear input–output model which is a practical alternative to the conventional linear counterpart. The model contemplates the possibility of different assumptions on returns to scale and is calibrated in a simple manner that closely resembles the usual technical coefficient calibration procedure. Multiplier calculations under this nonlinear version offer appropriate interval estimates that provide information on the effectiveness and variability of demand-driven induced changes in equilibrium magnitudes. In addition, and unlike linear multipliers, the nonlinear model allows us to distinguish between physical and cost effects, the reason being that the traditional dichotomy between the price and quantity equations of linear models no longer holds. We perform an empirical implementation of the nonlinear model using recent interindustry data for Brazil, China and United States. When evaluating the robustness of the derived marginal output multipliers and the induced cost effects under the nonlinear approach, the results indicate that marginal indicators in physical terms can be perfectly used to infer average impacts; this is not the case, however, for the derived cost effects where average measures are seen to be more adequate. At the computational level, the analysis proves the operational applicability of the nonlinear system while at the methodological level shows that scale effects are relevant in determining sectoral multipliers.

1. Introduction

There is a glaring contrast between the theoretical advances in nonlinear input–output (NIO) theory and the surprisingly scarce list of applications in the empirical literature. This divorce cannot be attributed to the computational requirements for solving nonlinear models. With today's specialized software computation should not be a decisive issue. The question probably lies on the informational requirements needed for the implementation of NIO models, particularly on sectoral response elasticities. As Lahiri (1983) acutely points out, empirical estimation of NIO models is nearly impossible—too many parameters to estimate given the available data observations. The same type of problematic data requirement situation is also common for the specification of computable general equilibrium (CGE) models but this has not stopped practitioners at all (see Dervis et al., 1982, Mansur and Whalley, 1984). CGE modeling and research has become a very important area for policy analysis and evaluation and this has been possible, in part, thanks to the adoption of operational assumptions on agents' behavior and the use of calibration techniques. We believe that practical implementation of NIO models is equally possible once we (i) accept some specific behavioral rules in the definition of production activities and (ii) are able to use observed empirical data for the parameterization of production processes.

The theory of NIO models has been concerned with establishing theorems that prove existence and uniqueness of solutions for a nonlinear version of the Leontief input–output (I–O) quantity equations. Under quite general conditions, but all of them sharing a modified system productivity assumption, existence and uniqueness can be proved. Sandberg (1973), Chander (1983), Fujimoto (1986), Szidarovszky (1989), and Herrero and Silva (1991), among others, provide the necessary theoretical background for NIO logical consistency. In a NIO model technical coefficients are not taken as fixed. Their variability can be attributed to many different factors (technical innovation, input substitution, productivity changes, non-homogeneity, etc.) as Rose (1983) very clearly explains in his review and assessment paper. Theorists need not concern themselves with these possibilities but applied economists should at least explore them and consider how to sensibly incorporate them. The nonlinearities we consider in this paper are of the scale-dependent type, i.e. changes in total output need not be proportional to changes in total inputs but still a unique production mix is all that is available to firms. In other words, isoquants are L-shaped but the isoquant map is not necessarily homothetic. Price-induced

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nonlinearities in coefficients due to smooth input substitution, as dealt with in Tokutsu (1989) or Sancho (2010), are not considered here where we focus on the role of scale effects. West and Gamage (2001), in turn, is one of the few empirical examples of using a nonlinear assumption although restricted to the households' income account, where average coefficients are substituted by marginal ones. Zhao et al. (2006) introduce a Cobb–Douglas production function for defining the interindustry technical coefficients but since their model does not contemplate any price behavior whatsoever, the selection of the input mix is very much based on some ad-hoc assumptions—such as maintaining total output constant when substitution takes place in some sector. This way of proceeding has little if any economic justification.

The paper follows this organization. Section 2 discusses the general characteristics of the proposed NIO model with scale effects. In Section 3 we undertake an empirical exercise with the proposed NIO model using 2011 interindustry data for Brazil, China and United States. Data is taken from the World I–O Database (WIOD). A Conclusion remarks section completes the paper.

2. Nonlinear input–output

2.1. Review of the conventional linear model

Interindustry data provide a detailed multisectoral depiction of the revenue–expenditure-output macroeconomic identities. Consider an economy composed of n distinct productive sectors indexed as $i, j = 1, 2, \ldots, n$. In the period when data is assembled, identified here by super index 0, the following identities representing the circular flow of income hold true for all $j = 1, 2, \ldots, n$:

$$\sum_{i=1}^{n} p_i^0 \cdot x_{ij}^0 + p_v^0 \cdot v_j^0 = \sum_{i=1}^{n} p_j^0 \cdot x_{ji}^0 + p_j^0 \cdot f_j^0 = p_j^0 \cdot x_j^0. \quad (1)$$

In expression (1) the left-hand side collects total expenditure in intermediate purchases and value-added acquisition incurred by sector j to carry out the production of its output x_j^0; the middle part is total revenue accruing to sector j from the sale of its output x_j^0 to other sectors – as intermediate demand – and to final demanders. Finally, the right-hand side of the expression is the value of total production x_j^0 obtained in sector j. Expression (1) can therefore be seen as a sort of sectoral budget constraint in terms of volume. Interindustry data, however, is expressed in value and the distinction between physical magnitudes $(x_{ij}^0, x_j^0, v_j^0, f_j^0)$ and prices – for goods and value-added – (p_j^0, p_v^0) is not usually available. We can take observed transaction values as if they were physical magnitudes and in doing so we redefine units in such a way that every one of the new units has a worth of 1 currency unit. In other words, we use new prices $p_j = 1$ for goods and $p_v = 1$ for value added so that $p_i \cdot x_{ij} = p_i^0 \cdot x_{ij}^0$, $p_v \cdot v_j = p_v^0 \cdot v_j^0$, and $p_j \cdot f_j = p_j^0 \cdot f_j^0$.

With this implicit normalization it is customary in interindustry analysis to omit the presence of prices in the balance identities in Eq. (1) for the base year. Contrary to what has been a common practice, for the time being we will keep them explicit for reasons that will become clear shortly. Consequently, from the expenditure perspective, identity (1) becomes:

$$\sum_{i=1}^{n} p_i \cdot x_{ij} + p_v \cdot v_j = p_j \cdot x_j \quad (1a)$$

while, from the revenue perspective, takes the following form:

$$\sum_{i=1}^{n} p_j \cdot x_{ji} + p_j \cdot f_j = p_j \cdot x_j. \quad (1b)$$

Notice that since only the price p_j is involved in Eq. (1b), it can be eliminated altogether from it if so desired. However written, expressions (1a)–(1b) are nothing but the representation of observed data. The standard I–O model adopts the assumption that input–output ratios and value-added ratios are constant; in other words, it takes output as proportional to inputs by way of assuming nonnegative technical coefficients defined by $a_{ij} = x_{ij}/x_j$ and $\upsilon_j = v_j/x_j$. In production theory terms, this technological relationship takes the form:

$$x_j = Min\left\{\frac{x_{1j}}{a_{1j}}, \ldots, \frac{x_{nj}}{a_{nj}}, \frac{v_j}{\upsilon_j}\right\}. \quad (3)$$

These coefficients are assumed to be unique and independent of the scale of production. Substituting these coefficients in Eq. (1a) and simplifying yields:

$$p_j = \sum_{i=1}^{n} p_i \cdot a_{ij} + p_v \cdot \upsilon_j \quad (4a)$$

which, translated into vector–matrix notation, can be expressed and solved as:

$$\mathbf{p}' = \mathbf{p}' \cdot \mathbf{A} + p_v \cdot \boldsymbol{\upsilon}' = p_v \cdot \boldsymbol{\upsilon}' \cdot (\mathbf{I} - \mathbf{A})^{-1} = p_v \cdot \boldsymbol{\upsilon}' \cdot \mathbf{L} \quad (4b)$$

provided that matrix $\mathbf{A}$, with $[\mathbf{A}]_{ij} = a_{ij}$, is productive and the value-added price p_v is taken as *numéraire*. Matrix $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$ in Eq. (5a) is the so-called Leontief inverse. Similarly, substituting in expression (1b) and eliminating now the "irrelevant" price p_j gives:

$$\sum_{i=1}^{n} a_{ji} \cdot x_i + f_j = x_j. \quad (5a)$$

In matrix terms we would obtain:

$$\mathbf{x} = \mathbf{A} \cdot \mathbf{x} + \mathbf{f} = [\mathbf{I} - \mathbf{A}]^{-1} \cdot \mathbf{f} = \mathbf{L} \cdot \mathbf{f}. \quad (5b)$$

The linear I–O model in expressions (4b) and (5b) is composed of two sets of equations, one for prices and one for quantities, which solve independently of each other. For a given technology matrix $\mathbf{A}$, cost covering prices $\mathbf{p}'$ depend only on the value-added technical coefficient vector $\boldsymbol{\upsilon}'$, while output levels $\mathbf{x}$ depend only on final demand levels $\mathbf{f}$. This is the well-known dichotomy between prices and quantities in the conventional I–O model and it is a property that derives from the linearity assumption in the technology.

2.2. A nonlinear input–output system

With the objective of describing the NIO model, the point of departure is a Leontief production function with no input substitution allowed. In the present formulation and in contrast with the standard case, however, output and inputs are no longer related through a linear relationship. Thus we posit that:

$$x_j = Min\left\{\frac{x_{1j}^{\beta_{1j}}}{a_{1j}}, \ldots, \frac{x_{nj}^{\beta_{nj}}}{a_{nj}}, \frac{v_j^{\beta_{vj}}}{\upsilon_j}\right\}. \quad (6)$$

Under the production technology in Eq. (6), the efficient locus becomes:

$$x_j = \alpha_{ij} \cdot x_{ij}^{\beta_{ij}} = \eta_j \cdot v_j^{\beta_{vj}} \quad (7)$$

with the notational change $\alpha_{ij} = 1/a_{ij}$ and $\eta_j = 1/\upsilon_j$. Clearly for $\alpha_{ij}, \beta_{ij}, \beta_{vj} > 0$, expression (7) represents a monotonically increasing and continuous production function, i.e. more output can only be obtained when