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# Dynamic behavior of product and stock markets with a varying degree of interaction

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ABSTRACT

We develop a macroeconomic behavioral model in order to analyze the interactions between product and financial markets. The real subsystem is represented by a simple Keynesian income–expenditure model, while the financial subsystem is represented by an equilibrium stock market with heterogeneous speculators, i.e., chartists and fundamentalists. The interactions between the two markets are modeled in the following way: the aggregate demand depends, among other variables, also on the stock market price, while the fundamental value used by speculators in their decisional process depends on the real sector economic conditions. In our model we introduce a parameter that represents the degree of interaction. With the aid of analytical and numerical tools we show that an increasing degree of interaction between markets tends to locally stabilize the system. This stabilization occurs via a sequence of period-halving bifurcations. Globally, we find that the stabilization process implies multistability, i.e., the coexistence of different kinds of attractors.



## 1. Introduction

Instabilities are known, both empirically and theoretically, to be features of all markets: the product markets, the labor market, and the financial markets.

Over the last twenty years, many stock market models have been proposed in order to study the dynamics of financial markets (see Hommes, 2013). According to such models, even in absence of stochastic shocks, the interaction between heterogeneous speculators accounts for the dynamics of financial markets. Those models, when endowed with stochastic shocks, are able to replicate some important phenomena, such as bubbles and crashes, excess volatility and volatility clustering. However, in this kind of models authors have restricted their attention to the representation and the dynamics of financial markets only and the existing feedbacks between the product and financial markets are completely neglected. An exception is represented, for instance, by Charpe et al. (2011), Lengnick and Wohltmann (2013), Scheffknecht and Geiger (2011), and Westerhoff (2012). Charpe et al. (2011) propose an integrated macro model, using a Tobin-like portfolio approach, and consider the interaction of heterogeneous agents in the financial market in order to obtain financial market instability. They find that unorthodox fiscal and monetary policies are able to stabilize unstable macroeconomies. Lengnick and Wohltmann (2013) propose an agent-based model with financial markets interconnected with a New Keynesian model with bounded rationality and explore the consequences of transaction taxes. The results are endogenous development of business cycles and stock price bubbles. Scheffknecht and Geiger (2011) present a financial market model with leverage-constrained, heterogeneous agents integrated with a New Keynesian standard model; all agents are assumed to be boundedly rational. They show that a systematic reaction by central bank on financial market developments dampens macroeconomic volatility. Finally, in Westerhoff (2012) the real sector is described via a Keynesian good market approach, while the set-up for the stock market includes heterogeneous speculators. More precisely, in Westerhoff (2012) the real sector is represented by an income–expenditure model in which expenditures depend also on the dynamics of the stock market price. On the other hand, the financial side is represented by a market where traders choose between two behavioral forecasting rules concerning the future development of the stock price: fundamentalism and chartism (Day and Huang, 1990; Hommes, 2013; Westerhoff and Wieland, 2010).[2] The stock market, in turn, is linked

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[2] Due to the variety of possible behavioral rules adopted by fundamentalists and especially by chartists, that is reflected by several different modeling choices, we decided to follow the line started by Day and Huang (1990) and recently reconsidered in Westerhoff and Wieland (2010) (see page 1138).

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to the good market since the stock market's fundamental value depends on national income. In Westerhoff (2012) the product market subsystem is described by a stable linear relation, while the financial sector is represented by a nonlinear relation, that is, by a cubic functional relation. In that way, the oscillating behavior is generated by the financial subsystem only. In Westerhoff (2012), it is shown that interactions between the real sector and the stock market appear to be destabilizing, giving rise to chaotic dynamics through bifurcations.

In our paper we present a model which is inspired to the one in Westerhoff (2012), but which also displays some crucial differences with respect to it. The first difference is that the oscillating behavior is generated by the real subsystem. To be more precise, the nonlinearity of the real subsystem is due to the nonlinearity of the adjustment mechanism of the good market with respect to the excess demand. Another difference with respect to Westerhoff (2012) is the way we represent and analyze the interaction between the two markets. We assume in fact that economic agents base their decisions on a weighted average between an exogenous value and an endogenous value given by the current realization of economic variables, such as stock prices and income. In this way, the parameter describing the weighted average represents also the degree of interaction between the two markets. The extreme values of the weighting parameter correspond to the two cases considered in Westerhoff (2012), i.e., the isolated markets case and the interacting markets scenario. The last main difference with respect to Westerhoff (2012) is given by our assumption that the financial market speed of adjustment tends to infinity, generating a permanent stock market equilibrium. Such assumption is motivated by the fact that the functioning of financial markets is such that the mechanism of adjustment of their prices is much faster than the mechanism of adjustment of good market prices. As a consequence of that equilibrium assumption, in our model national income and stock prices are jointly determined by a one-dimensional nonlinear map. Analytical and numerical tools are used in order to find the mechanisms and the channels through which instabilities are transmitted between markets.[3]

The main contribution of this paper to the existing literature is to focus on the role of feedback mechanisms by the real and financial sectors, not only for the dynamics and stability of a single market, but also for those of the economy as a whole. More precisely, our main finding, contrarily to Westerhoff (2012), is about the stabilizing role of an increasing degree of interaction. We believe such difference is due to the fact that, as explained above, in our model we assume that the speed of adjustment in the stock market approaches infinity, which implies that the stock market is always in equilibrium, while in Westerhoff (2012) the stock market may not be in equilibrium and therein a full market interaction decreases the stability parameter set.

The specific results that we obtain can be summarized as follows. We prove in Proposition 3.3 the existence of an absorbing interval attracting all forward orbits, which prevents the system from divergence. Moreover, we show the presence of chaotic dynamics in the sense of Li and Yorke (see Li and Yorke, 1975, and Proposition 4.2). Finally, with the aid of numerical tools, we show that an increasing degree of interaction between markets tends to locally stabilize the system. This stabilization occurs via a sequence of period-halving bifurcations. Globally, we find that the stabilization process implies multistability, i.e., the coexistence of different kinds of attractors.

The remainder of the paper is organized as follows. In Section 2 we introduce the model. In Section 3 we present analytical and numerical local results for both isolated and interacting markets. In Section 4 we analytically investigate the first flip bifurcation and the existence of Li–Yorke chaos and we numerically show the bifurcations leading from odd-period cycles to a chaotic regime. In Section 5 we present some global scenarios with multistability phenomena. Finally, in Section 6 we draw some conclusions and discuss our results.

[3] A Keynesian IS–LM model has recently been analyzed through modern dynamical system methods, such as averaging theory, in Guirao et al. (2012).

## 2. The model

### 2.1. The real sector

Similarly to Westerhoff (2012), we consider a model with a Keynesian good market, interacting with a stock market, in a closed economy with public intervention.

The dynamic behavior in the real sector is described by an adjustment mechanism depending on the excess demand. If aggregate excess demand is positive (negative), production increases (decreases), that is, income $Y_{t+1}$ in period $t+1$ is defined in the following way

$$Y_{t+1} = Y_t + \gamma g(Z_t - Y_t), \tag{2.1}$$

where $g$ is an increasing function with $g(0) = 0$, $Z_t$ is the aggregate demand in a closed economy, defined as

$$Z_t = C_t + I_t + G_t,$$

where $C$, $I$ and $G$ stand for consumption, investment and government expenditure, respectively, and $\gamma > 0$ is the product market speed of adjustment between demand and supply.

In order to conduct our analysis, denoting by $E_t = Z_t - Y_t$ the excess demand, we specify $g$ as the following sigmoid function

$$g(E_t) = a_2\left(\frac{a_1 + a_2}{a_1 e^{-E_t} + a_2} - 1\right),$$

with $a_1$ and $a_2$ positive parameters. With such a choice, $g$ is increasing and $g(0) = 0$. Moreover, it is bounded from below by $-a_2$ and from above by $a_1$. This prevents the real sector from diverging and thus creates a real sector oscillator. Indeed, the presence of the two asymptotes does not allow too large variations in income. We stress that this particular analytical specification does not compromise the generality of the achievements. In fact, we found analogous results for other sigmoid functions passing through the origin.

As commonly assumed, we suppose that private and government expenditures increase with national income. Moreover, like in Westerhoff (2012), page 3, we assume that the financial situation of households and firms depends on the performance of the stock market, too. In particular, private expenditure also increases with the belief about the stock price performance $\hat{P}_t$, defined as

$$\hat{P}_t = (1-\omega)\widetilde{P} + \omega P_t,$$

with $\omega \in [0,1]$, where $\widetilde{P}$ is the long-period fundamental value and $P_t$ the current stock price. $\hat{P}_t$ may be interpreted as a weighted average between the long-period fundamental value and the current stock price. In particular, when $\omega = 0$ the belief about the stock price performance is completely exogenous and coincides with the long-period fundamental value; this is the case of isolated markets considered in Westerhoff (2012). When instead $\omega = 1$ the belief about the stock price performance is completely endogenous and coincides with the current stock price; this is the case of interacting markets in Westerhoff (2012). On the basis of these considerations, we can write the relation between private and government expenditures and national income and stock price as

$$Z_t = C_t + I_t + G_t = a + bY_t + c\hat{P}_t = a + bY_t + c\left[(1-\omega)\widetilde{P} + \omega P_t\right], \tag{2.2}$$

where $a > 0$ defines autonomous expenditure, $b \in [0,1]$ is the marginal propensity to consume and invest from current income and $c \in [0,1]$ is the marginal propensity to consume and invest from current stock market wealth belief.