Economic Modelling 41 (2014) 281–288

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

Steady state distributions for models of locally explosive regimes: Existence and econometric implications

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ARTICLE INFO

Article history:
Accepted 17 March 2014
Available online xxxx

Keywords:
Bubbles
Asset prices
Steady state distributions
Non-linear time series
TAR models

ABSTRACT

The purpose of this paper is to examine the properties of locally explosive regimes in the light of steady state results for threshold auto-regressive (TAR) models recently derived by Knight and Satchell (2011) [Journal of Time Series Econometrics, 3]. We study the conditions under which a steady state distribution of deviations of asset prices from fair value can be obtained using our simple model based on our particular definition of a bubble, noting that it is applicable to locally explosive regimes. After deriving general results, the analysis is further extended by considering the steady state distribution in three cases of a normally distributed error process, a non normally (exponentially) distributed steady-state process and a switching random walk with a fairly general iid error process. Then, the issues related to unit root testing for the presence of bubbles using standard econometric procedures are examined. Our results shed light on the ubiquitous finding of no bubbles in the econometric literature.

1. Introduction

Financial bubbles seem to be a permanent and ongoing event in global financial markets. If bubbles are a characteristic of financial markets, we need to ask if such an occurrence is consistent with the notion of a steady state price distribution. We are unaware of any analytical research on this question. In the bubbles literature, deviations from fundamentals are modelled either as bubbles or fads which can induce switching behaviour. We can incorporate both these features in our definition of bubbles as locally explosive regimes. The purpose of our paper is to analyse the existence of a steady state distribution of price in the presence of bubbles. The model we choose is intentionally very simple so as to allow exact analysis. The exact analysis allows us to give conditions for the existence of, and derive closed form expressions for steady state distributions, means and variances. We provide precise conditions on the existence of the above for the Blanchard and Watson (1982) model. Critics may say that such a model fails to capture all features of macrodynamics. We agree, but note that such simplifications allow us to produce new results.

The point of our paper is to present a model of bubbles that allows for explosive behaviour in at least one regime but still allows for the existence of a steady-state distribution in asset prices, even when a bubble exists. This is in clear contrast to the cointegration literature whereby the existence of a bubble implies no steady state distribution. The implications of this are several. We can employ an empirical analysis, firstly, based on histograms and links between sample and population distribution functions. Secondly, our results shed new light on conventional tests for bubbles based on cointegration and this tendency to not find evidence of bubbles. Thirdly, it is possible to describe analytically such "bubble" distributions. We give three examples in Section 4. We are then able to analyse characteristics of the distribution such as leading terms, moments, when they exist, the dependence on regime probabilities inter alia.

Economists vary in their definitions of what a bubble is, however, many definitions are very similar to the one we quote below. Kindleberger (1989) defines a bubble as "upward price movement over an extended range that then implodes". He adds (Kindleberger, 1989), "a bubble is a sharp rise in the price of an asset or a range of assets in a continuous process, with the initial rise generating expectations of further rises and attracting new buyers — generally speculators, interested in profits from trading in the asset rather than its use or earning capacity". A consensus definition of a financial bubble might require the deviation of prices away from, and above, the fundamental value over an uncertain period of time. The existence of bubbles finds support in financial market experiences and even the most recent financial crisis.

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http://dx.doi.org/10.1016/j.econmod.2014.03.015
0264-9993/© 2014 Published by Elsevier B.V.

Authors like Azariadis and Guesnerie (1986), Azariadis and Smith (1998), Brunnermeier and Nagel (2004), Garber (2000), Krugman (2000), Poterba and Summers (1988) have documented the presence of bubbles and sunspots to explain financial market crashes.

An important stylized fact of asset prices is nonlinearity, which can be explained by the presence of regime switching. This nonlinear behaviour can be induced in a number of ways. We attempt here to identify a model for bubbles, using a new definition, by explicitly assuming a switching threshold autoregressive process for the price to capture the sudden collapse of a bubble. The key contention here for bubbles is that the trigger for the bubble's collapse is modelled by an exogenous sunspot process. Camerer (1989) defines a sunspot as an exogenous event that has two values for instance, 'sunspot' or 'no sunspot'. More generally, it can be thought of as an extrinsic random variable that does not influence the fundamentals of the economy. This is in line with a number of authors who try to capture the onset of a bubble by some purely exogenous event (like Azariadis and Guesnerie, 1986; Batini and Nelson, 2000; Bernanke and Gertler, 2000; Evans, 1991; Gali, 2011; Wang and Wen, 2011). This is similar to the famous example in the chaotic dynamics literature of "butterfly's wings".[1] Such an assumption allows us to use the results derived by Knight and Satchell (2011) to examine conditions under which there exists a steady state distribution of prices. The motivation of this class of models is, in our view, a plausible way to capture the statistical properties observable in the time series of asset prices.

Our explanation is closely related to the simple intuitive explanation of the mechanism behind bubble formation suggested by Shiller (2005): "If asset prices start to rise strongly, the success of some investors attracts public attention that fuels the spread of the enthusiasm for the market. New (often, less sophisticated) investors enter the market and bid up prices. This "irrational exuberance" heightens expectations of further price increases, as investors extrapolate recent price action far into the future. The market's meteoric rise is typically justified in the popular culture by some superficially plausible "new era" theory that validates the abandonment of traditional valuation metrics".

We introduce what we believe to be a new concept; that of a locally explosive model. We define this to be a threshold autoregression i.e., a regime-based model in which some but not all regimes may be explosive. As we shall see, such a model can still have a stationary distribution with finite moments. Similar ideas appear in Hall et al. (1999) except that they use a Markov switching model. Phillips et al. (2011) in contrast present a structure whereby the explosive behaviour of the model shrinks with the sample size. More details of locally explosive models are given in Section 3.

We also discuss the relevance of our results to testing for the presence of bubbles. Econometric tests based on cointegration techniques often rule out the existence of bubbles. Hall et al. (1999) point out that the testing strategy of investigating the cointegration properties of asset prices and observable underlying fundamentals, is based on the rationale that the existence of an explosive bubble would imply that prices are more explosive than the fundamentals. They also note that unit root and cointegration tests have little power to detect bubbles that collapse periodically. Our findings support Hall et al. (1999) in that one of the deficiencies of a cointegrating approach is that the stationarity properties of the data are analysed by testing the null hypothesis of a unit root in the levels and differences of the series against one-sided stationary alternatives rather than the more relevant explosive ones, an issue that Phillips et al. (2011) address. Although such tests should, in theory, be capable of revealing the existence of a rational bubble (since bubbles imply that differencing of prices will not be sufficient to induce stationarity), this is not an easy task as in small samples, series with explosive bubble components could look very much like stationary processes when differenced a sufficient number of times. Evans (1991) has criticized the use of cointegration techniques for testing the presence of bubbles by demonstrating how the presence of bubbles is often not detected in unit root tests. We add further evidence to illustrate why the null hypothesis of the presence of bubbles does not tend to receive enough statistical support.

As mentioned previously, the direction of our work is different from earlier literature in several respects. First, we intentionally keep the model conceptually as simple as possible. Thus we do not aim at this point to produce a model that can closely explain all of the observable statistical features of complex modern markets, but rather look for the simplest signature model of bubbles, perhaps the next order of approximation to reality after the random walk, which our approach encompasses. One motivation is that, even if not exhaustive, a simple model has a better chance of being capable of econometric estimation without over fitting. There are only two independent parameters in the model plus the choice of an error process, and we investigate the behaviour of the model across possible specifications. In the absence of a change in fundamentals, randomness is entirely responsible for igniting the bubble and causing the bubble to collapse. This means that the deterministic part of our dynamics does not suggest any typical time scales for these processes, making them essentially random, and similar to Poisson processes. Indeed, the bubble collapse (or ignition) is hard to predict.

Section 2 contains a discussion on the literature related to bubbles and testing their presence econometrically. In Section 3 we consider the existence of steady-state distributions and moments of prices for our model of bubbles. Section 4 looks further at steady-state distributions for prices by considering some explicit examples, either by specifying the error process or by reversing the question, and asking what error process will lead to a given distribution. In Section 5, we look at the implications of our results on the efficacy of conventional econometric tests for the presence of bubbles and Section 6 concludes.

2. Literature review

Under the literature for bubbles, one of the earlier models by Blanchard and Watson (1982) proposes a theory of rational bubbles in which agents' (rational) expectations are influenced in part by extrinsic random variables whose properties accord to historical bubble episodes. They consider a price process such that $p_t = \omega_t E_t(p_{t+1}) + D_t$ where $\omega_t = \frac{1}{1+r_t}$ (less than 1) is the rate of discount and D_t is an exogenous stationary dividend process and r_t is the rate of return. For algebraic simplicity and tractability, we can assume $r_t = r$ and hence $\omega_t = \omega$. They obtain a solution in which prices equal fundamentals, p^* (present discounted value of the dividend stream) by recursive substitution $p_t = \sum_{i=0}^{\infty} \omega^i E_t(D_{t+i}) = p^*$ under the transversality condition $\lim_{T\to\infty} E_t\left(\frac{1}{(1+r)^T} p_{T+1}\right) = 0$. They allow for solutions of the kind $p_t = p^* + c_t$, where c_t can be considered a bubble, under certain assumptions. Since then, there has been a lot of progress in dealing with the criticisms of their model and alternative models.

Another issue we recognise in this literature is determining the fundamental value of the asset. There are various methodologies used to address this, for instance Moinas and Pouget (2013) define the fundamental value of an asset as the price at which agents would be ready to buy the asset given that they cannot resell it later. Other approaches under rational bubbles include West (1983), Froot and Obstfeld (1991) and Santos and Woodford (1997). The recent survey chapter by Brunnermeier and Oehmke (2012) provides an excellent overview of relevant literature on bubbles. Evans (1991) constructs rational bubbles that periodically explode and collapse. More recent models make different assumptions about rationality and belief structures including Allen and Gorton (1993), Hong et al. (2006), Lansing (2010), Branch and Evans (2011) and Branch and Evans (2011). Brunnermeier

[1] Lorenz, E. (1972) Predictability: Does the Flap of a Butterfly's Wings in Brazil set off a Tornado in Texas? In a talk given at the meeting of the American Association for the Advancement of Science in Washington, D.C.