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The complexion of dynamic duopoly game with horizontal differentiated products

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ABSTRACT

In this paper, firms are considered on the hypothesis of having incomplete rationality expectation and incomplete information of the market to get the dynamic development of price competition behavior in the Hotelling model (Hotelling, 1929). Under the assumption of the heterogeneous expectations of two firms, we have observed that the Nash equilibrium price can be a dynamic equilibrium to realize when the speed of price adjustment is lower. However, the numerical simulation shows that the system may present a periodic and chaotic status when the speed of price adjustment is higher. The effect of the degree of horizontal differentiation on the stability of Nash equilibrium of the system is also discussed. We have a different conclusion from Lucino Fanti and Luca Gori's (2012); that is the greater the degree of product horizontal differentiation is, the more stable the Nash equilibrium of the system is.

1. Introduction

Oligopoly means that a few manufacturers produce the whole products and supply the whole market demands. Cournot (1838) was the first researcher to study an oligopoly model. Bertrand (1883) put forward another oligopoly model which was different from Cournot's. When oligopolists' price competition made equilibrium price status equal to marginal cost and firms have zero profit, it is similar to complete competition equilibrium. We called this the "Bertrand Paradox". Hotelling (1929) introduce product differentiation and solved this problem.

In many literatures, more scholars have studied homogeneous products in the Cournot model and Bertrand model. They have focused on chaotic phenomena with the speed of quantity or price adjustment. Kinds of incomplete rational expectations of naïve players, adaptive players and bounded rationality players were studied (Agiza, 1998; Agiza and Elsadany, 2003). Yassen and Agiza (2003), Peng et al. (2011) and Elsadany (2010) have studied the Cournot duopoly game of bounded rationality with delay. Delayed parameters have expanded the stability region of the system.

For the importance of the Cournot model and Bertrand model, the profits, products and consumer surplus in differentiated Cournot and Bertrand equilibrium were compared (Correa-López and Naylor, 2004; Fanti and Meccheri, 2011; Ghosh and Mitra, 2010).

If chaos means that the system is far away from equilibrium, it is evident that we can take measures to control this event. Ma and Mu (2007) and Holyst and Urbanowicz (2000) have demonstrated some chaotic phenomena and utilized the delayed feedback control to stabilize the chaos.

Generally speaking, chaos is considered as an irregular status. The whole stable system suffers from the collapse. However, some scholars have observed that chaos may be a better phenomenon. Matsumoto (2003), Wu et al. (2010) and Huang (2008)'s numerical simulation have demonstrated that firms could get higher profits in chaotic status than that of Nash equilibrium. Agliari et al. (2005) and Tramontana (2010) have analyzed flip or Neimark–Sacker bifurcation under a higher adjustment speed.

Their study is based on homogeneous products. However, in a realistic economy, it is too idealized to reach such a degree. Therefore, a study on differentiated products in a duopoly market is more popular. Singh and Vives (1984) have analyzed two differentiated duopoly markets with price and quantity competition. Häckner (2000) developed Singh and Vives' assumption and results: for more than two firms, price or quantity competition which was more efficient was not evident. Fanti and Gori (2012) have demonstrated that a higher degree of product differentiation made the market equilibrium instability.

In this paper, we investigate the effect of the degree of product differentiation on chaos in the Hotelling model with naïve expectation

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and bounded rationality. We have the conclusion that under some conditions, the greater the degree of product differentiation is, the more stable the Nash equilibrium of system is. This is different from Lucino Fanti and Luca Gori's conclusion (2012). The paper is organized as follows: Section 2 gives the hypothesis of the Hotelling model, Nash equilibrium and heterogeneous expectation of the system; Section 3 gives the theoretical analysis of the effect of the degree of product differentiation on the stability of the system. Section 4 gives the numerical simulation of bifurcation diagrams, strange attractors and sensitivity of initial values.

2. The model

2.1. Static Hotelling model of price competition

We focus on the effect of horizontal differentiation on the stability of the equilibrium in the dynamic system. We use the price competition part of the model which originated from the Hotelling product differentiation model, and it is called the static Hotelling price competition model. The model is as follows:

It is assumed that there are two firms, 1 and 2, whose product place values are a_1 and a_2 which are in the interval [0, 1], where $a_2 > a_1$. $\Delta a = a_2 - a_1$ gives the degree of product horizontal differentiation. “Place value” in this paper generally refers to horizontal characteristics of the brand, the color, the taste of the product, sale location, etc. where a_1 and a_2 are exogenous variables. One typical consumer's product preference h is subject to the uniform distribution of [0, 1], whose distribution function is $F(x) = P\{h \le x\} = x, x \in [0, 1]$. Roughly speaking, a consumer does not have a special favorite of every place value of [0, 1]. If a consumer chooses the product whose place value is a_1, besides the price p_1, he also needs to pay for a deviation cost $t(h - a_1)^2$. Similarly, if a consumer chooses the product whose place value is a_2, besides the price p_2, he also needs to pay for a deviation cost $t(h - a_2)^2$. Where t is called deviation cost rate. h^* is used to represent the indifference preference of place values a_1 and a_2, that is $p_1 + t(h^* - a_1)^2 = p_2 + t(a_2 - h^*)^2$, $h^* = \bar{a} + (p_2 - p_1)/2t\Delta a$ can be obtained, where $\bar{a} = (a_1 + a_2)/2$. 1/2 $t\Delta a$ is the competition intensity of the market. The smaller the degree of product differentiation Δa and the deviation cost rate t are, the more fierce the competition in the market is, and otherwise the weaker it is. The consumers' product demands are: $x_1(p_1, p_2; a_1, a_2) = P\{h \le h^*\} = h^*$, $x_2(p_1, p_2; a_1, a_2) = 1 - h^*$. We have the demand functions as follows:

$$\begin{cases} x_1(p_1, p_2; a_1, a_2) = \bar{a} + (p_2 - p_1)/2t\Delta a \\ x_2(p_1, p_2; a_1, a_2) = 1 - \bar{a} + (p_2 - p_1)/2t\Delta a. \end{cases} \tag{1}$$

It is assumed that the two firms' products have the same marginal cost $c > 0$. The two firms' profit functions:

$$\begin{cases} \pi_1(p_1, p_2; a_1, a_2) = (p_1 - c)(\bar{a} + (p_2 - p_1)/2t\Delta a) \\ \pi_2(p_1, p_2; a_1, a_2) = (p_2 - c)(1 - \bar{a} + (p_2 - p_1)/2t\Delta a). \end{cases} \tag{2}$$

It is clear that the two firms' marginal profits are:

$$\begin{cases} \pi_{1p_1} = \bar{a} + (p_2 - p_1)/2t\Delta a - (p_1 - c)/2t\Delta a \\ \pi_{2p_2} = 1 - \bar{a} + (p_1 - p_2)/2t\Delta a - (p_2 - c)/2t\Delta a. \end{cases} \tag{3}$$

The problem of firm $i (i = 1, 2)$ is under the condition of the given place values of a_1 and a_2, it chooses the price p_i to maximize $\pi_i(p_1, p_2; a_1, a_2)$. Let $\pi_{ip_i} = 0$, and the two firms' price reaction functions are:

$$\begin{cases} p_1^R(p_2) = (p_2 + c)/2 + t\bar{a}\Delta a \\ p_2^R(p_1) = (p_1 + c)/2 + t(1 - \bar{a})\Delta a. \end{cases} \tag{4}$$

The intersection of two reaction curves gives the Nash equilibrium:

$$\begin{cases} p_1^*(a_1, a_2) = c + 2t(1 + \bar{a})\Delta a/3 \\ p_2^*(a_1, a_2) = c + 2t(2 - \bar{a})\Delta a/3. \end{cases} \tag{5}$$

In the static Hotelling model with price competition, if the complete information of the market and complete rational behavior are satisfied, the Nash equilibrium can be obtained. However, it is hard to achieve the requirement. Thus, from a more practical point of view, it is assumed that firms do not have the complete rational expectation.

2.2. Dynamic Hotelling model of price competition with heterogeneous expectation

As is described above, we assume that firms do not have complete rational expectation and complete information of the market. In order to have a comparison with Fanti and Gori (2012)'s conclusion, we have the assumption of bounded rationality expectation of firm 1; that is firm 1 adjusts the next period price p_1' according to his own current price p_1, marginal profit π_{1p_1}, and the speed of price adjustment $v > 0$; that is $p_1' = p_1 + vp_1\pi_{1p_1}$. Firm 2 has a “naïve” bounded rationality expectation; he expects the price of firm 1 invariant the next period. Thus, he uses the optimal reaction function $p_2^R(p_1)$ as the next price;

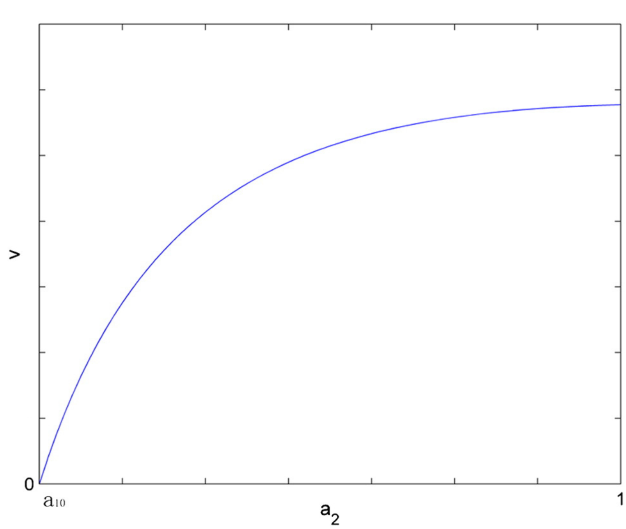

Fig. 1. Stability region of Hotelling model as function of a_2 when $\sqrt{3c/t} + a_1 \ge 1$.

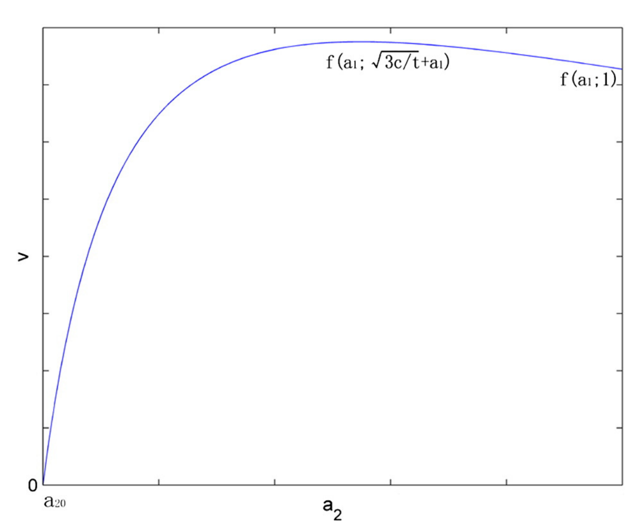

Fig. 2. Stability region of Hotelling model as function of a_2 when $\sqrt{3c/t} + a_1 < 1$.