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Regret theory and the competitive firm: A comment☆

Cuizhen Niu [a], Xu Guo [b,c], Tao Wang [d], Peirong Xu [e,*]

[a] School of Statistics, Renmin University of China, Beijing, China
[b] College of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing, China
[c] Department of Mathematics, Hong Kong Baptist University, Hong Kong, China
[d] Department of Biostatistics Yale School of Public Health, New Haven, USA
[e] Department of Mathematics, Southeast University, Nanjing, China

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ABSTRACT

In a recent paper, Wong [Wong, K. P. (2014), Regret theory and the competitive firm. Economic Modelling, 36, 172–175.] develops a model to examine the production behavior of a regret averse competitive firm. Wong discusses the sufficient condition to ensure the conventional result that the optimal output level under uncertainty is less than that under certainty hold. Our contributions in this note are two-fold. Firstly, we point out that Wong's condition in terms of the first order derivatives of the utility function and the regret function is actually not sufficient. Secondly and more importantly, we show that a sufficient condition should be in terms of the relatively increase rate of the first order derivatives of the two functions. That's, it's the ratio of the risk aversion and regret aversion degree that matters. Our proposed condition requests that the firm should be not too regret averse.

1. Introduction

Recently, the theory of regret aversion has gained a lot of attentions. This kind of behavioral characteristic is common and supported by a large body of experimental literatures and our life experience (see, e.g., Loomes and Sugden, 1987; Starmer and Sugden, 1993). To be precise, individuals and firms would avoid to make ex-post suboptimal decisions. For example, if the firms' prices turn out to be very high and sales turn out to be very good, firms might regret not producing more. Conversely, if prices turn out to be low and sales are poor, firms might regret an over-production. Bell (1982) and Loomes and Sugden (1982) propose a formal analysis of regret theory, which is further axiomatized by Sugden (1993) and extended to multiple choices by Quiggin (1994). In these works, regret is considered as the disutility of not having made the ex-post optimal decisions.

This theory has been adopted to considerable research in decision making under uncertainty. For instance, Braun and Muermann (2004) examine optimal insurance purchase decisions of individuals that exhibit behavior consistent with regret theory. Wong (2011) incorporates this theory to examine the optimal bank interest margin, the spread between the loan rate and the deposit rate of a bank. Wong (2012) further studies the behavior of a regret averse producer facing revenue risk and discusses the demand of insurance. Tsai (2012) investigates the bank's optimal loan rate under more stringent capital regulation.

In traditional economic analysis of competitive firm with uncertain output price (Broll, 1992; Sandmo, 1971; Viaene and Zilcha, 1998), the extant literature focuses on the risk averse firm whose preferences admit the standard von Neumann–Morgenstern expected utility representation. One noteworthy result in these literatures is that the risk averse firm optimally produces less under uncertainty than under certainty. When we incorporate the regret theory to study the production behavior of the competitive firm, it's theoretically and practically important to establish the relationship between the optimal output under uncertainty and under certainty. Paroush and Venezia (1979) firstly attempt to give an answer. Based on a bivariate utility function of profits and regret, they provide the conditions under which the optimal output under uncertainty is lower than that under certainty. However, their conditions depend on some endogenous variables and are not informative. In this direction, Wong (2014) further studies production behavior of a competitive firm under output price uncertainty when the firm is not only risk averse but also regret averse. By adopting the additive separable function which is proposed by Braun and Muermann (2004) and Muermann et al. (2006), they re-examine the impact of regret on the firm's production decision as compared with the case of certainty. The most important result of Wong (2014) is that they provide a sufficient condition under which the regret-averse firm's optimal output level under uncertainty is less than that under certainty. They show

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* Corresponding author. Tel.: +86 186 025 52090590.
E-mail address: xupeirong@seu.edu.cn (P. Xu).

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that it's the ratio of the first order derivatives of the utility function and the regret function that determines the production behavior.

Our contributions in this note are two-fold. Firstly, we point out that Wong's condition is in fact not sufficient to obtain the conventional result of the extent literature that the optimal output level under uncertainty is less than under certainty. Then how to correct his condition to get a meaningful result? This leads to the second contribution of the note: a sufficient condition should be in terms of the relatively increase rate of the first order derivatives of the utility function and the regret function. This actually is the ratio of the risk aversion and regret aversion degree. This adds a novel result in the literature of the competitive firm under uncertainty.

The rest of the paper is organized as follows. In Section 2, we introduce the model of a competitive firm under not only risk aversion but also regret aversion. Then we discuss the condition given by Wong (2014). In Section 3 we present our new sufficient condition under which the regret averse firm's optimal output level under uncertainty is less than that under certainty. The final section concludes.

2. The model

To make this note to be understood easily, we first briefly introduce the model. Consider a competitive firm which produces a single commodity with random per-unit price $\tilde{P}$ and according to a special cost function $C(Q)$. Here $Q \geq 0$ is the output level, and $C(\cdot)$ satisfies that $C(0) = C'(0) = 0$, $C'(\cdot) > 0$ and $C''(\cdot) > 0$.[1] The support of $\tilde{P}$ is $[\underline{P}, \overline{P}]$ with $0<\underline{P}<\overline{P}<\infty$. The firm's final profit is given by $\tilde{\Pi} = \tilde{P}Q - C(Q)$. To account for the regret that ex-post suboptimal decisions have been made, Wong (2014) introduces the following bivariate utility function[2]:

$$V(\Pi, \Pi^{\max} - \Pi) = U(\Pi) - \beta G(\Pi^{\max} - \Pi),$$

here Π^{max} is the maximum profit that the firm could have earned if the realized output price is known in advance. Further if we have observed the realized output price P, $\Pi^{\max}$ would take the form that $\Pi^{\max}(P) = PQ(P) - C[Q(P)]$ with $C'[Q(P)] = P$. $U(\cdot)$ is a von Neumann–Morgenstern utility function with $U'(\cdot) > 0$ and $U''(\cdot) < 0$ accounting for the firm's risk aversion. While $G(\cdot)$ is a regret function such that $G(0) = 0$, $G'(\cdot) > 0$ and $G''(\cdot) > 0$.[3] The parameter $\beta \geq 0$ is a constant regret coefficient, indicating the extent of the regret aversion.

As a result, the production decision problem of the competitive firm reads:

$$\max_{Q \geq 0} E\left\{U\left[\Pi\left(\tilde{P}\right)\right] - \beta G\left[\Pi^{\max}\left(\tilde{P}\right) - \Pi\left(\tilde{P}\right)\right]\right\},$$

here $E(\cdot)$ is the expectation operator with respect to the cumulative distribution function, $F(P)$, of the random output price $\tilde{P}$.

The first-order condition is then given by:

$$E\left\{\left\{U'\left[\Pi^*\left(\tilde{P}\right)\right] + \beta G'\left[\Pi^{\max}\left(\tilde{P}\right) - \Pi^*\left(\tilde{P}\right)\right]\right\}\left[\tilde{P} - C'(Q^*)\right]\right\}.$$

where an asterisk (*) indicates an optimal level. To make comparison, denote the optimal output level by Q^n when the uncertain output price, $\tilde{P}$, is set to be its expected value, $E\left(\tilde{P}\right)$. In this case, we can have $C'(Q^n) = E\left(\tilde{P}\right)$. Further define $\Pi^n(P) = PQ^n - C(Q^n)$. For more details about this model, see Wong (2014).

The main result in Section 3 of Wong (2014) is stated in the following proposition:

Proposition 1. *If $U'''(\cdot) \geq 0$ and $G'''(\cdot) \geq 0$, then a sufficient condition that ensures the regret-averse firm to produce less than the optimal output level under certainty, i.e., $Q^* < Q^n$, is that the constant regret coefficient, β, is sufficiently small such that*

$$\beta \leq \frac{U'\left\{\Pi^n\left[E\left(\tilde{P}\right)\right]\right\} - U'\left[\Pi^n\left(\overline{P}\right)\right]}{G'\left[\Pi^{\max}\left(\overline{P}\right) - \Pi^n\left(\overline{P}\right)\right] - G'(0)}. \tag{6}$$

Let $\Psi(P) = U'[\Pi^n(P)] + \beta G'[\Pi^{max}(P) - \Pi^n(P)]$. From Wong (2014), we know that to obtain $Q^* < Q^n$, we only need to show that $E\left\{\left\{\Psi\left(\tilde{P}\right) - \Psi\left[E\left(\tilde{P}\right)\right]\right\}\left[\tilde{P} - E\left(\tilde{P}\right)\right]\right\}$ is negative. For all $P<E\left(\tilde{P}\right)$, we can have that $\Psi(P)>\Psi\left[E\left(\tilde{P}\right)\right]$ under some mild conditions. However, for all $P>E\left(\tilde{P}\right)$, we need some extra conditions to get $\Psi(P)<\Psi\left[E\left(\tilde{P}\right)\right]$. In Wong (2014), he states that 'Since $\Psi(P)$ is strictly convex in P and $\Psi'\left[E\left(\tilde{P}\right)\right]<0$, it follows from condition (6) that $\Psi(P)<\Psi\left[E\left(\tilde{P}\right)\right]$ for all $P>E\left(\tilde{P}\right)$'. This statement is not true. The convexity of $\Psi(P)$ can imply that $\Psi'(P)>\Psi'\left[E\left(\tilde{P}\right)\right]$ for all $P>E\left(\tilde{P}\right)$. However, it's still not clear about the sign of $\Psi'(P)$ for $P>E\left(\tilde{P}\right)$ since $\Psi'\left[E\left(\tilde{P}\right)\right]$ is negative. From the condition (6) and the definition of $\Psi(P)$, we can only obtain that $\Psi\left(\overline{P}\right)<\Psi\left[E\left(\tilde{P}\right)\right]$. Consider a special value $\overline{P}>P^*>E\left(\tilde{P}\right)$. Under condition (6), we can get that

$$\begin{aligned}
&\Psi(P^*) - \Psi\left[E\left(\tilde{P}\right)\right] \\
&\leq U'\left[\Pi^n(P^*)\right] - U'\left\{\Pi^n\left[E\left(\tilde{P}\right)\right]\right\} \\
&\quad + \left(U'\left\{\Pi^n\left[E\left(\tilde{P}\right)\right]\right\} - U'\left[\Pi^n\left(\overline{P}\right)\right]\right)\frac{G'\left[\Pi^{\max}(P^*) - \Pi^n(P^*)\right] - G'(0)}{G'\left[\Pi^{\max}\left(\overline{P}\right) - \Pi^n\left(\overline{P}\right)\right] - G'(0)} \\
&= \left(U'\left\{\Pi^n\left[E\left(\tilde{P}\right)\right]\right\} - U'\left[\Pi^n(P^*)\right]\right) \\
&\quad \times \left(\frac{U'\left\{\Pi^n\left[E\left(\tilde{P}\right)\right]\right\} - U'\left[\Pi^n\left(\overline{P}\right)\right]}{U'\left\{\Pi^n\left[E\left(\tilde{P}\right)\right]\right\} - U'\left[\Pi^n(P^*)\right]}\frac{G'\left[\Pi^{\max}(P^*) - \Pi^n(P^*)\right] - G'(0)}{G'\left[\Pi^{\max}\left(\overline{P}\right) - \Pi^n\left(\overline{P}\right)\right] - G'(0)} - 1\right).
\end{aligned}$$

It's easy to know that $\Pi^n\left[E\left(\tilde{P}\right)\right]<\Pi^n(P^*)<\Pi^n\left(\overline{P}\right)$, thus we can have $U'\left\{\Pi^n\left[E\left(\tilde{P}\right)\right]\right\}>U'\left[\Pi^n(P^*)\right]>U'\left[\Pi^n\left(\overline{P}\right)\right]$. This results in that $U'\left\{\Pi^n\left[E\left(\tilde{P}\right)\right]\right\} - U'\left[\Pi^n(P^*)\right]>0$ and

$$\frac{U'\left\{\Pi^n\left[E\left(\tilde{P}\right)\right]\right\} - U'\left[\Pi^n\left(\overline{P}\right)\right]}{U'\left\{\Pi^n\left[E\left(\tilde{P}\right)\right]\right\} - U'\left[\Pi^n(P^*)\right]}>1.$$

On the other hand, note that $\Pi^{\max}(P^*) - \Pi^n(P^*)<\Pi^{\max}\left(\overline{P}\right) - \Pi^n\left(\overline{P}\right)$ and $G''(\cdot) > 0$, thus we can get:

$$\frac{G'\left[\Pi^{\max}(P^*) - \Pi^n(P^*)\right] - G'(0)}{G'\left[\Pi^{\max}\left(\overline{P}\right) - \Pi^n\left(\overline{P}\right)\right] - G'(0)}<1.$$

Consequently, it's difficult to determine the sign of $\Psi(P^*) - \Psi\left[E\left(\tilde{P}\right)\right]$ by using the above approach.

To obtain that for all $P>E\left(\tilde{P}\right)$, $\Psi(P)<\Psi\left[E\left(\tilde{P}\right)\right]$, some strong conditions are needed, such as

$$\beta \leq \frac{U'\left\{\Pi^n\left[E\left(\tilde{P}\right)\right]\right\} - U'\left[\Pi^n(P)\right]}{G'\left[\Pi^{\max}(P) - \Pi^n(P)\right] - G'(0)}, \quad P>E\left(\tilde{P}\right).$$

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[1] Same to Wong (2014), the strict convexity of the cost function is assumed. This assumption reflects the fact that the firm's production technology exhibits decreasing returns to scale. See also Broll et al. (2006).

[2] Our notations are borrowed from Wong (2014). Braun and Muermann (2004) and Muermann et al. (2006) also use similar utility function.

[3] This assumption is taken from Wong (2014) which indicates that the more pleasurable the consequence that might have been, the more regret will be experienced.