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Optimal retirement age, leisure and consumption

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ABSTRACT

In this paper, we study the determination of optimal retirement age, optimal leisure time, and optimal consumption, and we also analyze their relationships using an optimal control theory. We establish a life cycle model and analyze the factors of consumption, leisure, saving, mortality and retirement behaviors simultaneously with an orthogonal-array experimental design. Our results show that the initial salary level and the growth rate of salary are the most important determining factors of the optimal retirement age. The initial consumption level and the interest rate are also important factors affecting optimal retirement age. The mortality improvement has a minor effect on the optimal retirement age. The effects of the Social Security on the optimal retirement age depend on the Social Security tax and the level of Social Security benefit.

1. Introduction

Population aging is a common phenomenon in the member countries of Organization for Economic Co-operation and Development (OECD) because of mortality improvement and lower fertility, and the actual retirement ages are lower than compulsory retirement age in most of these countries. Retirement planning is therefore an increasingly important process for every individual. It requires answers to certain crucial questions. For example, how much is enough to retire comfortably? How does one determine the optimal retirement age? What are the key factors to be considered in the retirement decision? All of these questions are concerns of individuals, enterprises and government. Forman and Chen (2008) discuss the optimal retirement age qualitatively from three different perspectives: the government, the enterprises, and the individuals. In this paper we discuss the optimal retirement age quantitatively from the individual perspective. We use the optimal control theory by constructing a life cycle model to determine the optimal retirement age, optimal leisure time and optimal consumption, and analyze their relationship and the main factors affecting the optimal retirement age.

How do individuals decide when to retire? Retirement research indicates that some individuals make rational decisions about the appropriate age of retirement, but many may not be so rational. Choosing an optimal and "economically feasible retirement age" is a complicated endeavor (Forman and Chen, 2008). Commonly considered factors affecting the retirement age are consumption patterns, life span, initial wealth, wage level, interest rate, parameters of risk aversion, preference of leisure, pension benefits and the Social Security benefits. Other factors have also been considered. Some evidence indicates that retirement decision is linked to health conditions (see, for example, Sickles and Taubman, 1986). However things may have changed dramatically in the 1990s. As pointed out by Forman and Chen (2008): "The American work has changed over the years. As our economy has changed from manufacturing to services, physical strength has become less important and computer and people skills become more important." Working longer is not only just possible, it also becomes a reality. In our model, we assume that the health conditions allow people to work longer. Another factor is taxation of income (see Cremer et al., 2004). Increasing retirement age will increase saving and total wealth, but decrease leisure time, and vice versa. Therefore, the tradeoff between the utility gain from early retirement and the reduction of retirement benefit plays a vital role in determining the optimal retirement age. There are several papers discussing this issue. Fields and Mitchell (1984) find workers who have more income would retire earlier, and those who would have more wealth gain from working would postpone retirement, using a life cycle model and a survey data, considering the influence of the structures of earnings, Social Security, and pension benefits. Wells (1987) establishes an objective function of maximizing the total discounted utility earned before retirement plus the discounted utility of wealth accumulated at the retirement time, considering the leisure preference during working life and assuming a monomial function of time. The paper focuses on the optimal allocation of time to work activities and optimal selection of retirement age. However, the horizon in the discussion is 0 to retirement age. It is also known that consumption after retirement is also an important factor influencing the retirement age. Bloom et al. (2011b) assume that the total expected utility is a function of discounted utility

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of consumption minus the discounted disutility of working over the life time assuming that retirement is caused by poor health conditions at old age. Their results show that improvements in health and longevity would disproportionally increase retirement age, but the wealth effects generated by the compound interest would lead to early retirement and lower savings. Bodie et al. (2004) discuss the consumption, labor/leisure and portfolio choice in life cycle, with labor income flexibility and habit formation, but do not consider mortality. Gustman and Steinmeier (2005) also use a life cycle model and find that 5% of the population would delay retirement if the early entitlement age in the U.S. was raised. Laitner and Silverman (2005) focus on consumption behaviors after retirement and argue that a drop in consumption is not consistent with the predictions of life-cycle models. Farhi and Panageas (2007) study optimal consumption and portfolio choice in a framework where investors adjust their labor supply through an irreversible choice of their retirement time. Tucker (2009) discusses optimal retirement age with two different decision criteria: maximizing the present value at age 62, and optimizing the level of retirement income. They also compare the optimal retirement ages between these two decision criteria. Kalemli-Ozcan and Weil (2010) set up an objective function of maximizing the discounted utility of consumption over lifetime plus the discounted utility of leisure over the time from retirement to death. They consider the effects of the different mortality rates at different ages on the optimal solutions. The differences of our paper from theirs are threefold. First, we revise the objective function by considering the utility of leisure over the whole lifetime. Although Wells (1987) also consider the leisure preference during a person's working life, we use log function of leisure preference instead of monomial expression. It should be noted that the coefficients of monomial expression are difficult to be accurately estimated, since leisure preference is subjective. From this perspective, we improve both Wells (1987) and Kalemli-Ozcan and Weil (2010). Giarini (2012) notes: "What is really changing is that the notions of old age themselves. Taking into consideration of the ability of each individual to be autonomous (in physical and/or mental terms), many studies and surveys indicate that the average 60 or even 80 years old today equates a person about 15 to 20 years younger living one century or more ago. Statistics based not on age, but on physical capability, indicates in fact that in many countries, the population is not 'ageing' but 'rejuvenating'. In fact, we live in a 'counter-aging society'". In this situation, a lot of people would rather like to increase leisure during their working years and delay the retirement age so as to improve their life quality. Therefore, it is very important to consider how leisure affects retirement age. Additionally, we assume that the salary is a function of time instead of a constant level assumed by Kalemli-Ozcan and Weil (2010). We also consider the influence of Social Security on optimal retirement age. Finally, we consider the relationship among the optimal retirement age, leisure, and consumption, and discuss which factors are most important in determining the retirement age. Usually, it is difficult to conduct a sensitivity analysis when considering several factors that may change simultaneously. Especially if the number of factors considered is rather large, as this will result in unrealistically lengthy calculations. In this article, we do the sensitivity analysis with an orthogonal-array experimental design which is used in engineering studies (Taguchi and Konishi, 1987), so that we can consider several factors such as consumption, leisure, saving, mortality, income, and the degree of risk aversion of consumers, simultaneously.

The conventional wisdom attributes the tendency towards early retirement to Social Security benefits and pension benefits. In particular, the Social Security early entitlement age — 62 years old, seems to play an important role in an individual's retirement decisions. In the work of Crawford and Lilien (1981), the income effects (early retirement) and substitution effects (postponement of retirement) between Social Security and pension plans make their functions in the retirement decisions ambiguous. Costa (1998) claims that those factors may influence retirement patterns, but they should not be the determinants for retirement rates, especially for those individuals over the age of 64. In consideration of the arguments that Social Security is a factor in the trend towards earlier retirement, we construct the models with Social Security and without Social Security.

The remainder of our paper is organized as follows: In Section 2, we establish the models without considering Social Security and study the most important factors affecting optimal retirement age. Section 3 discusses the effects of Social Security on optimal retirement age. Section 4 concludes the paper.

2. Models without considering social security

Assume that an individual's life span is T, the initial age of entering work is a_0, and the individual can choose to retire at time R between a_0 and T. Assume that C_t is the consumption at time t, h_t is the proportion of leisure at time t, bounded between 0 and 1, so that $(1 - h_t)$ is the proportion of work at time t. Then $h_t < 1$, when $a_0 \le t \le R$ and $h_t = 1$, when $R < t < T$. Assume that $f(t)$ is the earning rate at time t and retirement is a one-time irrevocable decision. In other words, if the agent retires at time t, he/she cannot re-enter the labor market thereafter. Assume that the force of mortality at time t is $\lambda(t)$, the probability of being alive at age t is $e^{-\lambda(t)t}$, the interest rate is r, and the discount rate is p.

The objective of the agent is to maximize the discounted value of the utilities of consumption and leisure over the lifetime, that is,

$$\text{Max} \int_{a_0}^{\infty} [u(C(t)) + v(h_t)] e^{-(\lambda(t)+p)t} dt \tag{1}$$

subject to a life-time budget constraint:

$$\frac{dW}{dt} = (1-h_t)f(t) - C(t) + (r + \lambda(t))W_t \tag{2}$$

where W_t is the wealth of an individual at time t. The Hamiltonian equation can beconstructed as follows:

$$H(t) = [u(C(t) + v(h_t)] e^{-(\lambda(t)+p)t} + \phi[(1-h(t))f(t) - C(t) + (r + \lambda(t))W_t]. \tag{3}$$

The following are the first-order conditions for an optimal C:

$$\frac{d\phi}{dt} = -\frac{\partial H}{\partial W} = -\phi(r + \lambda(t)) \tag{4}$$

$$\frac{dH}{dC} = u'(C(t)) e^{-(p+\lambda(t))t} - \phi = 0. \tag{5}$$

From Eq. (4), we can easily know that

$$\phi = e^{-(r+\lambda(t))t + c_1} \tag{6}$$

where c_1 is a constant. By combining Eqs. (3) and (6), we can obtain the following equation:

$$\begin{aligned} H(t) = {} & [u(C(t) + v(h_t)] e^{-(\lambda(t)+p)t} \\ & + e^{-(r+\lambda(t))t + c_1} [(1-h_t)f(t) - C(t) + (r + \lambda(t))W_t]. \end{aligned} \tag{7}$$

The optimal value of h_t may be obtained by maximizing $H(t)$ with respect to h_t. Taking the partial derivative of H with respect to h, we obtain

$$\frac{\partial H}{\partial h} = v'(h_t) e^{-(\lambda(t)+p)t} - f(t) e^{-(r+\lambda(t))t + c_1}. \tag{8}$$