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Approximate Whittle analysis of fractional cointegration and the stock market synchronization issue ☆

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ABSTRACT

I consider a bivariate stationary fractional cointegration system and I propose a quasi-maximum likelihood estimator based on the Whittle analysis of the joint spectral density of the regressor and errors. This allows to estimate jointly all parameters of interest of the model. I lead a Monte Carlo experiment to investigate the finite sample properties of this estimator when integration orders are less than 1/2. However, it is not so easy for practitioners to identify whether or not the observed time series are stationary. This issue is investigated by extending the numerical analysis to mean-reverting non-stationary region of the parameter space, although the proposed estimator is not theoretically designed to handle this case. The results display good finite sample properties in both cases, stationary and non-stationary. Thereby, it reveals that making a wrong decision on the stationarity of raw series does not lead to an erroneous conclusion. An application to the stock market synchronization is proposed to illustrate the empirical relevance of this estimator.

1. Introduction

Consider the following triangular bivariate fractional cointegration representation

$$(1-L)^{\gamma}(y_t-\beta x_t)=\varepsilon_{1t},\quad (1-L)^{\delta}x_t=\varepsilon_{2t},\quad t=1,2,\ldots,n, \tag{1}$$

where $\delta\in(0,1/2)$, $\gamma\in[0,1/2)$ and $(1-L)^{\alpha}$ is the fractional filter, further denoted Δ^{α} and defined by its binomial expansion

$$(1-L)^{\alpha}=\sum_{k=0}^{+\infty}a_k(\alpha)L^k,\quad a_k(\alpha)=\frac{\Gamma(k-\alpha)}{\Gamma(k+1)\Gamma(-\alpha)} \tag{2}$$

$$\Gamma(z)=\int_0^{+\infty}t^{z-1}e^{-t}dt, \tag{3}$$

where L is the lag operator. Following the general and seminal definition of the cointegration proffered by Granger (1986), in Eq. (1), y_t is said cointegrated if the error term $\nu_t=y_t-\beta x_t$, satisfies $\nu_t\sim I(\gamma)$ with $\gamma<\delta$ and $x_t\sim I(\delta)$. When $\gamma\geq\delta$, the regression is spurious, whether y_t and x_t are stationary or not, as long as their orders of integration sum up to a value greater than 1/2 (see Tsay and Chung, 2000). Many studies restrict their analysis to integer integration orders. In most cases, x_t is assumed to possess a unit root (i.e. $\delta=1$) and the traditional cointegration also imposes $\gamma=0$. When δ is assumed equal to unity and $v_t\sim I(\gamma)$, $0\leq\gamma<1$, the estimation of Eq. (1) usually proceeds in two steps (see the pioneer work of Cheung and Lai, 1993). The first step is to estimate the long run coefficient β, and the second step is to estimate γ, the long memory parameter of the residuals.

More recently, the idea that cointegration relationship can exist between stationary variables (i.e. $\delta<1/2$) has emerged. Some studies have suggested adapted estimators for β. For instance, Robinson (1994) develops a consistent semi-parametric narrow-band least squares estimator (NBLS) of β that essentially performs the time domain least square estimator (LSE) on a degenerating band of frequencies around the origin. Then, Christensen and Nielsen (2006) demonstrate the asymptotic normality of the NBLS when $\delta+\gamma<1/2$ and Nielsen and Frederiksen (2011) extend it to the weak fractional cointegration[1] (i.e. $\delta-\gamma<1/2$). In most studies, these estimators are combined with semi-parametric estimators of long memory (see for instance Christensen and Nielsen, 2006; Marinucci and Robinson, 2001; Nielsen and Frederiksen, 2011). Among others, we can mention the well-known log-periodogram regression (LPE) of Geweke and Porter-Hudak (1983), Künsch (1987), Robinson (1995) or Andrews and Guggenberger (2003). Velasco (2003) suggests to estimate simultaneously δ and γ but requires a consistent estimator of β. Earlier, a similar proposition was made by Sowell (1989) to estimate jointly β, δ and γ in the time domain, and applied

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[1] The terms weak and strong fractional cointegration mean here the intensity with which deviations from the long-run equilibrium will disappear.

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by Dueker and Startz (1998). Nielsen (2007) extended this innovative approach to the frequency domain by suggesting a local Whittle quasi-maximum likelihood analysis of the joint spectral density of x_t and v_t (hereafter locale QMLE). Simultaneously, Hualde and Robinson (2007) developed a $\sqrt{n}$-consistent parametric estimator of weak fractional cointegration following the suggestion of Robinson and Hualde (2003) and exploiting an error correction form of (1). Their estimator has the advantage of covering a wide range of integration orders; however, it requires a more complex optimization procedure than Nielsen (2007).

In this paper, I propose a full-band Whittle analysis of a stationary fractional cointegration model (hereafter FC-QMLE). This approach contrasts with that of Nielsen (2007) because of the spectral density function that embeds the parametric form the short-and long-term dynamics, although the two likelihood functions are very similar. The two methods have their advantages and disadvantages. For instance, as mentioned in Nielsen (2007), a fully parametric approach is more efficient, using the entire sample, but is inconsistent if the parametric model is misspecified while a semi-parametric approach is invariant to any short-term dynamics but at the cost of higher variance. Indeed, the local FC-QMLE is only $\sqrt{m}$-consistent where m depends on the bandwidth selection.[2] A Monte Carlo experiment documents the finite sample properties of the FC-QMLE. A first part of the simulation is consistent with the model (1) and the parameter space of δ and γ, while the second is not. Indeed, in practice, it is difficult to know whether or not the integration order of the time series is less than 1/2 and a wrong decision concerning the stationarity of the series can lead to misuse the estimator. To investigate this issue, one part of the Monte Carlo experiment consists of estimating a mean-reverting non-stationary model (i.e. $\delta<1$), although the FC-QMLE is not designed to handle this case. In both cases, the finite sample properties are good. The main result of this numerical analysis lies that a wrong decision on the stationarity of raw series does not lead to an erroneous conclusion concerning the existence of a long run relationship. Some insights are put forward regarding this result.

The remainder of the paper is laid out as follows. In Section 2 some generalities on the long memory are exposed and the FC-QMLE is developed. In Section 3 the Monte Carlo simulation is described and performed. In Section 4, an application on the stock market synchronization is proposed to illustrate the empirical relevance of the FC-QMLE. Section 5 concludes the paper.

2. The stationary fractional cointegration model

In this section, the model is developed with respect to the stationary regions of long memory parameters (i.e. δ and γ less than 1/2). The development below first introduces the Whittle estimator outside the cointegration context. Later, I take into account the cointegration framework by considering a multivariate framework.

2.1. The univariate Whittle estimator of fractional integration

In a more general form than the Eq. (2), the Gaussian process x_t (equivalently v_t, also assumed to be Gaussian) can be restated as follows

$$x_t=\sum_{j=0}^{\infty}\kappa(j;\delta)\varepsilon_{1t-j},\quad \sum_{j=0}^{\infty}\kappa(j;\delta)^2<\infty,\quad \kappa(0;\delta)=1,\quad t\in\mathbb{Z} \tag{4}$$

for $\kappa(.;\delta)\in\mathbb{R}$. In order to state the spectral density of x_t, let $g(\lambda;\delta)=\sum_{j=0}^{\infty}\kappa(j;\delta)e^{ij\lambda}$ be the transfer function. Accordingly, the spectral density of x_t is defined by,

$$f_x(\lambda;\delta)=\frac{\sigma^2}{2\pi}\left|g(\lambda;\delta)\right|^2,\quad \sigma^2=\mathrm{var}(\varepsilon_{2t}), \tag{5}$$

where $|g(.)|$ is the complex modulus of $g(.)$. Assuming that σ^2 depends on δ, the well-known Whittle estimator (QMLE) of δ is therefore defined as $\hat{\delta}_n=arg\,min_{\delta\in D}\,Q_n(\delta)$ with D a compact subset of $\mathbb{R}$ and

$$Q_n(\delta)=-n\left[\int_{-\pi}^{\pi}\log f_x\left(\lambda_j;\delta\right)d\lambda+\int_{-\pi}^{\pi}fx\left(\lambda_j;\delta\right)^{-1}I_{xx}\left(\lambda_j\right)d\lambda\right], \tag{6}$$

where $\int_{-\pi}^{\pi}logf_x(\lambda_j;\delta)d\lambda<\infty$ is assumed, implying that x_t is non-deterministic. In Eq. (6), $I_{xx}(\lambda_j)$ denotes the periodogram of x_t defined as

$$I_{xx}\left(\lambda_j\right)=w_x\left(\lambda_j\right)\overline{w}_x\left(\lambda_j\right),\quad w_x\left(\lambda_j\right)=\frac{1}{2\pi n}\sum_{t=1}^{n}x_te^{it\lambda_j}, \tag{7}$$

where $w_x(\lambda_j)$ is the Fourier transform of x_t and $\lambda_j=(2\pi j/n)$ is the angular frequency. In the following, it will be preferred the discrete version of (6) which is

$$Q_n(\delta)=-\sum_{j=1}^{n}\left[log\,\mathrm{f}_x\left(\lambda_j;\delta\right)+f_x\left(\lambda_j;\delta\right)^{-1}I_{xx}\left(\lambda_j\right)\right] \tag{8}$$

Notice that in $Q_n(\delta)$ the zero frequency is left out of the summation, implying that this estimator is invariant to the presence of non-zero mean. Minimize Q_n approximates the normal log-likelihood up to a constant-order term. Moreover, under some regularity conditions, it is conjectured that $\hat{\delta}$ is asymptotically efficient in the Fisher sense and asymptotically normally distributed (see Fox and Taqqu, 1986).

2.2. The multivariate Whittle estimator of fractional cointegration

One can attempt to estimate step by step the Eq. (1) using a consistent estimator of β. However, it should be more efficient to estimate jointly β, δ and γ. This alternative is widely discussed in Robinson and Hualde (2003) and applied in the time domain and the frequency domain by Hualde and Robinson (2007) and Nielsen (2007), respectively. In this subsection, the model has no short-term dynamics (see Subsection 2.3 for a version with short-term dynamics). Thereby, the likelihood function I consider is closely related to the likelihood function of the local FC-QMLE. However, in contrast to the narrow band approach of Nielsen (2007), I consider the whole sample. A similar development is conducted by Hosoya (1997), concerningthe Whittle estimator of multivariate ARFIMA processes (multivariate QMLE) but does not deal with the cointegration framework.

Let $\mathbf{w}_t$ be a bivariate Gaussian sequence such as $\mathbf{w}_t=(\nu_t,x_t)'$, $\theta_1=(\gamma,\delta)'$ and $\theta_2=(\beta)$. Rewriting Eq. (4) in a multivariate framework, we obtain

$$\mathbf{w}_t=\sum_{j=0}^{\infty}K(j;\theta_1)\varepsilon_{t-j},\quad \sum_{j=0}^{\infty}\mathrm{tr}\,K(j;\theta_1)\Sigma(\varsigma)K(j;\theta_1)'<\infty,\quad K(0;\delta)=I,\quad t\in\mathbb{Z} \tag{9}$$

where elements of K are all real, $\varepsilon_t=(\varepsilon_{1t},\varepsilon_{2t})$ and Σ is a real, symmetric and positive definite 2×2 matrix whose elements are $\varsigma=(\Sigma_{11},\Sigma_{12},\Sigma_{22})'$. It follows the transfer matrix $G(\lambda;\theta_1)=\sum_{j=0}^{\infty}K(j;\theta_1)e^{ij\lambda}$ and the joint spectral density function

$$f_{\mathbf{w}}(\lambda;\theta_1,\varsigma)=\frac{1}{2\pi}G(\lambda;\theta_1)^{-1}\Sigma(\varsigma)\left(G(\lambda;\theta_1)^{-1}\right)^{*}, \tag{10}$$

[2] For practical purpose, the bandwidth selection is important because of the variance of the estimator that can increase. For instance, a too high bandwidth can deteriorate the variance if the process possesses a short run dynamics because of the confusion in the frequency domain between the low and high frequencies.