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Long-term research and development incentives in a dynamic Cournot duopoly

Yee Jiun Yap [a],*, Shravan Luckraz [b], Siew Kian Tey [a]

[a] Institute of Mathematical Sciences, Faculty of Science, University of Malaya, 50603 Kuala Lumpur, Malaysia
[b] School of Economics, Faculty of Social Sciences, University of Nottingham Ningbo China, 199 Taikang East Road, Ningbo, 315100, China

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ABSTRACT

This paper constructs an ex-ante asymmetric R&D Cournot differential game with knowledge spillovers. It shows that in the long-run equilibrium firms have incentives to innovate as long as the knowledge externalities are bi-directional. We also carry out a series of numerical simulations of the differential game to illustrate our results.

1. Introduction

We develop a differential R&D game played by ex-ante asymmetric firms and in which the dynamics of technological diffusion depends on the technology gap between the firms. It has been well established that when one firm independently develops a cost reducing innovation, the firm's competitors benefit in the sense that they can use the innovation to reduce their own costs. When such spillover effects are significant, noncooperative firms might be expected to research too little from the standpoint of the industry since each firm tends to ignore the positive externality which its research generates on the cost of its rival firm (see D'Aspremont and Jacquemin, 1988; Henriques, 1990; Simpson and Vonortas, 1994). However, when spillovers are endogenous it is also observed that the firm's disincentive to engage in R&D activity is partially offset because its own R&D can potentially enhance its capacity to absorb its rival's technology (see Grunfeld, 2003; Kamien and Zang, 2000; Katsoulacos and Ulph, 1998; Kultti and Takalo, 1998). Moreover, reduced costs of rival firms due to spillovers will lead all firms to compete more intensively in the product market. Empirical findings by Cohen and Levinthal (1989) reinforce the fact that spillovers have two opposing effects on R&D investment in strategic games: firstly, they increase the firm's incentive to raise its own R&D and, secondly, they create a disincentive for the rival firm to invest in R&D as free riding becomes a better strategy.

Our approach relates to the R&D game literature in Industrial Organization (IO). In fact, an important strand in the IO literature argues that process spillovers play a key role in R&D games. D'Aspremont and Jacquemin (1988, 1990) and Kamien et al. (1992) have independently developed game theoretical models to analyze both the cooperative and noncooperative behaviors of firms that engage in R&D activities when spillovers exist. While subsequent research by Henriques (1990), Suzumura (1992), Salant and Shaffer (1998), Simpson and Vonortas (1994), Amir (2000) and many others have extended and generalized their models, very few studies have emphasized on the explicit modeling of spillovers in R&D games. The lack of attention given to the treatment of spillovers can be regarded as a lacuna in this literature as empirical works by Cohen and Levinthal (1989) and Griliches (1992) clearly point out both the complexity and importance of spillovers in R&D models. In fact, Cohen and Levinthal (1989) show that contrary to conventional wisdom, intra-industry spillovers can encourage R&D investment. Moreover, Cameron (1999) observed that as the technology gap between a leader firm and the follower firm narrows, the follower must undertake more formal R&D since its ability to freeride on the leader's R&D reduces. Hence, spillovers are not completely exogenous as assumed in the R&D game literature; they depend on the technology gap between firms. Our paper aims to take this relationship between spillovers and technology gap into account.

Katsoulacos and Ulph (1998) were the first to endogenize spillovers in the two stage R&D game. In contrast to previous works which considered the spillover rate as purely exogenous when comparing the cooperative case with the noncooperative regime, they focus on the impact of research joint ventures on innovative performance. The concept of endogenous spillovers is explored further by Kamien and Zang (2000)

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* Corresponding author.
E-mail addresses: yjiun@um.edu.my (Y.J. Yap), Shravan.Luckraz@nottingham.edu.cn (S. Luckraz), skteysmile@gmail.com (S.K. Tey).

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and generalized by Leahy and Neary (2007) who argue that the firm cannot capture any spillovers from its rival without engaging in R&D itself. By incorporating absorptive capacity as a strategic variable, they distinguish between two components of spillovers; an exogenous component which represents involuntary spillovers from the firm's R&D activity and an endogenous component that allows the firm to exert control over spillovers. Our notion of spillovers is more general than the one used by these authors as it not only allows for absorptive capacity but also allows the spillover to depend on the technology gap between firms.

Our proposed framework uses the strategic interaction approach of R&D games to develop a dynamic Cournot duopoly model in which firms can invest in process innovations in an environment with imitation via knowledge spillovers. Time is assumed to be continuous and while firms still choose R&D before output as in D'Aspremont and Jacquemin (1988), a differential equation is used to describe how the spillover function (which determines the rate of technology diffusion) evolves over time. We ask whether R&D incentives can be sustained in an environment where technological innovation is almost a public good. We prove the existence of two types of asymmetric equilibria; one in which the leader maintains its technological advantage and one in which the follower catches up with the leader. We find that if the technology diffusion is bidirectional, the equilibrium where both firms invest in R&D at a constant positive rate is stable. Hence, we conclude that the imitation via knowledge spillovers does not deter innovation. While our results are similar to those by Spence (1984) and Bessen and Maskin (2009), our framework differs from theirs as we do not assume that the firms are symmetric (as in Spence, 1984), the technology diffusion rate in our model is not exogenous (as in both Bessen and Maskin, 2009; Spence, 1984) and dynamic strategic interactions with feedback effects are taken into account in our model (unlike Bessen and Maskin, 2009; Spence, 1984).

A precursor paper by Luckraz (2008) considers a similar framework as ours in the context of endogenous growth theory. Our model differs from the model in Luckraz (2008) in the following ways. First, in contrast to Luckraz (2008), here both technology catch-up and leapfrogging are allowed. Secondly, we find some important properties of the steady-state equilibrium that Luckraz (2008) was unable to find and finally, unlike Luckraz (2008) we are able to draw more direct conclusions about whether imitation via knowledge spillovers can hinder R&D.

The rest of the paper is organized as follows. In Section 2 we present the dynamic Cournot R&D game. In Section 3 we present our results and Section 4 contains some brief concluding remarks. The proofs are presented in Appendix A.

2. The model

We assume that the market structure is a duopoly in which at each time t, Firm 1 and Firm 2 produce an identical product and compete in Cournot fashion in the product market. Our Cournot assumption comes from the fact that we are interested in modeling cost reducing innovations rather than product innovation; hence, we assume product homogeneity just like in D'Aspremont and Jacquemin (1988). The game proceeds as follows. In each time t the two firms play a two stage Cournot game. Firms conduct process R&D to reduce their per unit cost of production at the first stage and choose output in the second stage. Each firm's marginal cost of production evolves over time according to an equation of motion. Time is assumed to be continuous. While we assume that one firm is the technology leader and the other firm is the laggard, we do not assume as in Luckraz (2008) that the leader is always more productive than the follower. In fact, we impose such a restriction only at $t = 0$ and hence, technology catch-up and leapfrogging are allowed in this model. Moreover, we assume that the technology leader also benefits from some minimal spillovers from the follower but to a lesser extent than which the follower benefits from the leader.

More formally we denote time by $t \in [0, +\infty)$ and assume that for each $t \in [0, +\infty)$, Firm 1 and Firm 2 face a demand function given by $P_t = A / Q_t$, where $q_{1t} + q_{2t} = Q_t$.[1] In order for our demand function and its corresponding welfare function to be well-defined, we need to assume that both price and quantity are bounded. In particular, we assume that there exist $\overline{P}$ and $\overline{Q}$ such that $Q_t \in \left[\frac{A}{\overline{P}}, \overline{Q}\right]$ and $P_t \in \left[\frac{A}{\overline{Q}}, \overline{P}\right]$.

The marginal cost of production of firm i is given by c_{it} and there are no fixed costs. We assume that firms can invest in R&D to reduce their marginal cost of production. More formally, we assume that for each i, c_{it} is given by

$$C_{it} \equiv \frac{1}{X_{it}} \tag{1}$$

where X_{it} is the productivity level of firm i. We assume that for each t and $i = 1, 2$, $X_{it} \in [1, +\infty)$. The time derivative of firm i's productivity level is given by

$$\dot{X}_{it} = \Lambda_{it}\left(X_{it}, X_{jt}\right) R_{it} \tag{2}$$

where X_{i0} is given, $X_{10} > X_{20} > 1$ and $R_{it} \in [0, +\infty)$ is the level of R&D conducted by firm i in time t. Moreover, Firm 1 is the technology leader and Firm 2 is the technology follower.[2] We assume that the depreciation rate is zero for simplicity. We also assume that $\Lambda_{it}(X_{it}, X_{jt}){:}[1, +\infty)^2 \to \mathbb{R}_+$ is given by

$$\Lambda_{it}\left(X_{it}, X_{jt}\right) \equiv X_{it}^{1-\sigma_i} X_{jt}^{\sigma_i} \tag{3}$$

where $i = 1, 2$, $i \neq j$ and $0 < \sigma_1 \leq \sigma_2 < 1/2$ is the technology diffusion parameter.σ_i plays a crucial part in our model as it reflects the extent to which technological knowledge is a public good in the model. Note that the technology leakage is involuntary and there is imitation via knowledge spillovers for innovations. On the other hand, each firm needs to undertake some R&D on its own in order to benefit from the technology transfer. While we assume an ex-ante asymmetric setup in which the follower can always freeride on the leader at least as much as the leader can free-ride on the follower, our range of parameter values for σ_1 and σ_2 also allows us to consider extreme cases like $\sigma_1 \to 0$ and $\sigma_2 \to 1/2$ (the laggard firm fully free-rides on the leader) or $\sigma_1 \to \sigma_2$. The ex-ante asymmetric assumption allows us to determine whether, with the imitation via knowledge spillovers, the follower will choose a very low level of innovation in equilibrium while freeriding on the leader's R&D. σ_i is assumed to be less than 1/2 to reflect the fact that the elasticity of firm i's productivity with respect to its own R&D is greater than the elasticity of its productivity with respect to its rival's R&D. Thus, the technology diffusion process is imperfect.

Our definition of spillovers is similar to Cohen and Levinthal (1989) together with some extensions. In particular we define spillovers to include valuable knowledge generated in the research process of the leader and which becomes accessible to the follower if and only if the latter is reverse engineering the innovator's research process. It is important here to note that empirical findings by Cohen and Levinthal (1989) state that spillovers have two opposing effects on R&D investment in strategic games: firstly, they increase the firm's incentive to raise its own R&D and, secondly, they create a disincentive for the rival firm to invest in R&D as free riding becomes a better strategy.

In practical terms, our assumption that $\sigma_1 \leq \sigma_2$ will imply that when an industry's market leader is surpassed by the follower, the rate of technological diffusion from the new leader (old follower) to the new

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[1] Note that Novshek's (1985) assumptions are too strong to capture this demand function. In fact, it can be shown that weaker assumptions are needed to capture demands with constant elasticity. Our demand specification and Cobb Douglas spillover function are more tractable mathematically. For detailed discussions of these functional forms and their implications see Luckraz (2011). For an example with linear demand and linear costs, see Luckraz (2007).

[2] Note that $X_{10} > X_{20}$ implies that the leader is more productive than the follower at $t = 0$.