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Higher order expectations in sentiment asset pricing model

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ABSTRACT

In the spirit of beauty contests, we study the effect of higher order expectations on sentiment asset pricing. The sentiment asset pricing model with higher order expectations shows that, in general the higher sentiment causes the higher price, but, higher order expectations contribute to smoother price path and defend the impact of sentiment. Regarding the problem of taking higher order or first order, the investors with second order can survive in a specific area where sentiment is rather optimistic or pessimistic and investors with first order expectations are the majority.

1. Introduction

As early as Keynes (1936) introduced the influential metaphor of financial markets as a beauty contest, it has been recognized that, when facing uncertainty, investors needed to consider not just an understanding of expectations about the fundamentals of asset, but also an understanding of action about other market participants' expectations, and an understanding of higher order expectations. Most asset pricing literatures have focused primarily on the former and ignored the effect of mass psychology (see Brown and Jennings, 1989; Grossman, 1976; Mendel and Shleifer, 2012). Higher order expectations mean that investors form expectations of other investors' expectations of an asset's payoffs. In the words of Keynes, investors are concerned, not with what an investment is really worth to a man who buys it for keep, but with what the market will value it.

Naturally, the higher order expectations should play an important role in asset pricing model. Unfortunately, though a number of authors adopt different ways to model how diverse investors' assessments affect the asset pricing, the role of higher order expectations has been neglected for decades until recently.

Following the insight of beauty contest, lots of recent literatures state the important role of higher order expectations in asset pricing. Allen et al. (2006) study the role of higher order expectations in asset pricing and demonstrate the failure of the law of iterated expectations, the result does not support the average expectations, and so higher order expectations differ from average expectations of the asset's payoffs. From then on, researches of higher order expectations begin to rise. Banerjee et al. (2009) discuss how higher order expectations can explain price drift in stock market and show it is necessary to generate price drift for heterogeneous beliefs. Kondor (2012) points out that the reason of forming higher order expectations is that early investors have to guess the information of investors acting later and public information polarized higher-order expectations without polarizing first order expectations. One of the main conclusions of these literatures is that expectations with higher order will generate a marked difference comparing to beliefs with the first order.

Moreover, the traditional asset pricing theory also suggests that investors behave rationally and won't make irrational error. Arguably, this requires an unrealistic degree of sophistication on the part of market participants. For example, it seems unlikely that investors are uncertain about fundamentals and correctly estimate or filter their information infected from some irrational factors in a strongly bullish or bearish market. The study of irrationality of the investor had largely been neglected for decades, but fortunately, it is now receiving significant attention in the growing field of behavioral finance. Behavioral finance argues that the investment strategy may be impacted by investor noise, investor psychology, and investor sentiment. De Long et al. (1990) show us how irrational investors with misperception affect prices. Daniel et al. (1998) study how two kinds of well-known psychological biases (overconfidence and biased self-attribution) cause the asymmetric shifts on asset revenue. However, these kinds of abstract biases model cannot provide quantitatively empirical analysis. Instead, we find that investor sentiment model can fill this gap. Moreover,

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after examining the role of higher order expectation in asset prices, Bacchetta and Van Wincoop (2008) state that errors of market beliefs can be caused by deviations from rationality, such as investor's overconfidence and market psychology. So we turn to study some sentiment asset pricing models that have been developed to emphasize the significant role of investor sentiment in asset pricing model, such as Yang and Yan (2011), Yang and Li (2013), Yang and Zhang (2013a,b).

However, none of these sentiment models consider the higher order expectations and study how higher order expectations affect the sentiment asset pricing. In this paper, we will give a powerful demonstration and show higher sentiment makes higher price, but, higher order expectations contribute to smoother price path in the condition of different sentiments.

Through the model with two kinds of investors with different order expectations, we illustrate that investors with the second order can survive in a specific area where the market sentiment is rather optimistic or pessimistic and investors with first order expectation are the majority in the market.

The remainder of this paper proceeds as follows. In Section 2, we present a traditional rational model with higher order expectations. Section 3 deduces the sentiment model with higher order expectations. Section 4 considers a sentiment model with two different order expectations and we will discuss the advantage of investors with second order expectations. Section 5 concludes the paper. Proofs and derivations are in the appendices.

2. Rational model with higher order expectations

To develop the intuition of how higher order expectations affect the role of sentiment in the sentiment model, we start with a rational model with higher order expectations.

There is a continuum of investors, who trade a risky asset with liquidation value θ against a safe asset with liquidation value of one, of unit measures indexed by i. There are three periods—0, 1 and 2. In period 2, the risky asset will be liquidated, where the liquidation price is θ. As a convention, we take $P=\theta$. In period 0, all investors share the same initial public information: the liquidation value θ of risky asset is distributed normally with mean P_0 and variance $1/\alpha$, where P_0 is also the initial price of the risky asset, and α describes the precision of the public information.

Before entering into trading, each investor may also rationally observe a private signal about θ which satisfies: $v_i^R=\theta+\varepsilon_i$, where ε_i is a normally distributed noise term with 0 and variance $1/\beta$, and the noise terms $\{\varepsilon_i\}$ are i.i.d. across investor i. No other source of information can be acquired by the investors, and, the private signal of each investor is not observable by others. Based on private signal and public signal, investor i forms his expectation of θ. Similar with Morris and Shin (2002), we defined that $E_i(\theta)$ stands that investor i estimates value on θ by depending on the information set he has, $\overline{E}(\theta)=\int_0^1 E_i(\theta)di$ is the average expectation of all the investors, and $E_i^2(\theta)=E_i\left[\overline{E}(\theta)\right]$ stands for expectation of investor i who cares about the average expectations and we call $E_i^2(\theta)$ second order expectations, $\overline{E}^2(\theta)$ stands for the average expectations of second order expectations, hence, $E_i^k[\theta]$ stands for the k order beliefs of investor i, and $\overline{E}_i^k[\theta]$ is the average of k order expectations. All the above calculations adopt the Bayesian information updating rule.

Suppose that all investors share the same exponential utility function $u(W_{2i})=-e^{-\gamma W_{2i}}$ and defined only on the wealth in the period 2. The parameter γ is the absolute risk aversion, and we can consider the reciprocal of it as the investors' risk tolerance.

Now all the investors live for maximizing the utility and thus we can obtain the demand equation. Facing a current price P_1 for the risky asset with the initial wealth W_0, investor i chooses his demand x_i to maximize the expectation of utility of wealth, which is

$$x_i=\arg\max E_i[-\exp[-\gamma(W_{2i})].\tag{1}$$

Maximizing this expression is equivalent to minimizing minus this expression, which is in turn equivalent to minimizing the log of that. Assuming for the moment that θ is normally distributed conditional on the information set, investor i adopts the k order expectations to evaluate θ, the detailed process of deduction will be seen in Appendix A, the demand equation can be given as follows:

$$x_i=\frac{\left[E_i^k(\theta)-P_1\right]}{\gamma var_i^k(\theta)},\tag{2}$$

where $var_i^k(\theta)$ denotes the variance of $\overline{E}_i^{k-1}(\theta)$ conditional on information set of investor i, which can be given below:

$$E_i^k(\theta)=\left(1-\frac{\beta^k}{(\alpha+\beta)^k}\right)P_0+\frac{\beta^k}{(\alpha+\beta)^k}v_i^R,$$
$$var_i^k(\theta)=\left(\frac{\beta}{\alpha+\beta}\right)^{2k-2}[var_i(\theta)]=\frac{\beta^{2k-2}}{(\alpha+\beta)^{2k-1}}.$$

Then the average demand of all investors is given by

$$\overline{X}=\frac{\left[\overline{E}_i^k(\theta)-P_1\right]}{\gamma var_i^k(\theta)},$$
$$\text{where }\ \overline{E}_i^k(\theta)=\left(1-\frac{\beta^k}{(\alpha+\beta)^k}\right)P_0+\frac{\beta^k}{(\alpha+\beta)^k}\theta.$$

Considering that the short term exogenous net supply of the asset is fixed and equal to 0. Imposing market clearing, then we can get $\overline{X}=0$, and rearranging gives the price equation as follows:

$$P_1^*=\overline{E}^k(\theta).\tag{3}$$

Observe that $\frac{\beta}{\alpha+\beta}\in(0,1)$, therefore, the equilibrium price in a model of higher order expectations will approach more slowly to the liquidation value of the asset. In other words, the higher order beliefs amplify the effect of the initial price on the equilibrium price. It implies that this model can describe momentum or price drift, where the traditional asset pricing theory suggests (see Allen et al., 2006; Banerjee et al., 2009).

For tractability, we assume that $P_0=0,\overline{X}=0$ in the rest of paper, and then the result of equilibrium price in the equation only stands for the relative path of P_0. In other words, the positive or negative value of equilibrium price illustrates the tendency of upward or downward. Thus we can rewrite Eq. (3) as follows:

$$P_1^*=\frac{\beta^k}{(\alpha+\beta)^k}\theta.\tag{4}$$

In period 1, the investors trade the asset at price $P^*{}_1$. It is consistent with Kyle et al. (2013) and Grossman (1976). This points out that the equilibrium price will reveal the average of all investors' information.

However, the disadvantage of this model is obvious. Eq. (4) shows that the path of P_1^* simply floats between the value of P_0 and θ, but in the realistic financial market, the price path is always just like a random walk. So we should incorporate other factors to make our model more persuasive and explanatory. In the next section we turn to the sentiment pricing model with higher order expectations.

3. Sentiment model with higher order expectations

A number of rational expectation models adopt the information structure which the noise term of private information can be canceled out after summing up (see Allen et al., 2006; Bacchetta and Van Wincoop, 2008; Grossman, 1976). However, De Long et al. (1990)