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Collective behavior and options volatility smile: An agent-based explanation

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ABSTRACT

This paper represents an initial effect to shed light on the determinants of option implied volatility smile from the micro perspective of traders' behavior. We compare the zero intelligence behavior and the collective behavior with the agent-based simulation. We find that the constant implied volatility, which is the assumption of the Black–Scholes model, can be obtained under the environment of the zero intelligence traders; while the smile shape of implied volatility, which is more consistent with the practical option market worldwide, can be explained by traders' collective behavior. Moreover, different degrees of collective behavior are tested to result that with the increasing of collective degree the implied volatility curve becomes steeper.

1. Introduction

1.1. Options volatility smile

It is well known that the most prominent anomaly in option pricing is the volatility smile. This stylized fact demonstrates the shape of a smile when you plot the implied volatility as a function of exercise price calculated from the observed market options prices. This phenomenon means higher values of the implied volatility are taken at the deep in-the-money (ITM) or deep out-of-the-money (OTM) than at-the-money (ATM). However, there is a conflict between this empirical discovery and the classical Black and Scholes (1973) option pricing theory. According to the Black–Scholes model, we should obtain a horizontal straight line which implied that any options for buying or selling the same underlying stock with the same expiration date, but with different exercise prices, should have the same implied volatility. Obviously, this is not the case in the real option markets.

Various models have been proposed so far to explain this anomaly. Popular ones include stochastic volatility models, jump-diffusion models, heterogeneous belief models, adaptive expectation models, risk aversion models and limits to arbitrage models. Based on the stochastic volatility models, Renault and Touzi (1996) exhibited a symmetric smile locally centered on the current forward price. To provide more flexible modeling of the time variation in the smirk and the volatility team structure, a two-factor stochastic volatility model which can generate stochastic correlation between volatility and stock return was proposed by Christoffersen et al. (2009). Regarding the jump-diffusion models, Bates (1996) pointed out that jump fear can better explain the smile than stochastic volatility models. Kou (2002) proposed a double exponential jump-diffusion model to produce analytical solutions for a variety of option pricing problems and give an explanation to the volatility smile. Pan (2002) captured both stochastic volatility and jumps in an arbitrage free model to examine the joint time series of the S&P 500 index and near the money option prices. They found that jump-risk premia become more prominent during volatile markets and concluded that the form of jump-risk premia is impotent to explain the volatility smirk. Owing to fluctuations in the financial markets from time to time, Xu et al. (2009) presented a fuzzy normal jump-diffusion model for European option pricing, with the uncertainty of both randomness and fuzziness in the jumps to understand the option pricing anomalies. Recently, a jump-to-default extend LRJ model with positive correlated stochastic volatility was examined to show the best effects in fitting market price and generating reasonable positive volatility skews Bao et al. (2012). On heterogeneous belief models, Ziegler (2002) showed the smile effect caused by heterogeneous in investors' beliefs. Meanwhile, Buraschi and Jiltsov (2006) presented that a model that takes information heterogeneity into account can explain the dynamics of option volume and the smile better than can reduced-form models with stochastic volatility. In a different perspective, David and Veronesi (2002) and Guidolin and Timmermann (2003) explain the implied volatility smile based on a Bayesian learning mechanism. Their models assume that investors alter their expectation due to change in fundamentals and parameter uncertainty. Another model

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also examined the role of adaptive expectation in the formation of observed implied volatility smile (Chalamandaris and Rompolis, 2012). Moreover, multi-agent simulations show that heterogeneity of traders' beliefs and the way traders update their expectations have significant effects both on equilibrium prices and on the emergence of the implied volatility smile (Qiu et al., 2012; Vagnani, 2009). In the aspect of risk aversion, an equilibrium analysis was made to display that stock market crash can change the risk attitude with which market participants view the index option (Bates, 2008). Meanwhile, an agent-based prospect theoretical model was proposed to demonstrate that the loss aversion feature of traders is capable of explaining the stylized fact of implied volatility (Suzuki et al., 2009). On the theory of limits to arbitrage, Rubinstein (1994) related the smile effect to market imperfections and frictions, such as transaction costs, short selling, taxes and other trading restrictions, which lead only to band in which option prices should lie, rather than a single arbitrage-free price. Pena et al. (1999) employed calls and puts traded on the IBEX-35 Index to conclude that transaction costs expressed by bid–ask spread is a key determinant of the curvature of the volatility smile.

Although we recognized the undisputed contribution of these studies on the volatility smile, we must observe that these researches either focus on the macro modeling variable exogenously given, such as a volatility process, a jump-diffusion process and transaction costs; or on the micro individual variable, such as heterogeneous beliefs and individual risk aversion. The pricing processes of the underlying assets in these models have not included the collective behavior prevailing on the financial markets. Inspired by the work of Han (2008), which examined whether investor sentiment about the stock market affects prices of the S&P 500 options, we analyze the effect of crowds in terms of a change of degree of collective behavior in our model.

1.2. Collective behavior in financial markets

Financial markets can be considered as complex systems having many interacting elements. In complex physical systems, interactions between constituents cause “collective modes” having special statistical properties which reflect the underlying dynamics. For example, during the spreading panic of market crashes, most agents sell their stocks. This supports the empirical finding that large price swings occur when the preponderance of trades have the same buy/sell decision (Gabaix et al., 2003). Peron and Rodrigues (2011) also verified that during a financial crisis, a synchronous state emerges in the system, defining the market's direction, and the distance between stocks tends to shorten indicating a collective dynamics. The size-effect and the periodic decisions of market heterogeneous participation are also found to explain the collective behavior (Egenter et al., 1999; Sato and Holyst, 2008). Similarly, agents learn from each other and tend to adopt the strategy that gives the most payoffs (Duffy and Feltovich, 1999). Given the price patterns at any point in time, a few of the most profitable technical strategies dominate the market because every technical trader wants to maximize his/her profit by using the most profitable strategy copied from each other. This phenomenon can also be described as opinion convergence or herding behavior. Cont and Bouchaud (2000) presented a simple model of a stock market where a random communication structure between agents generically gives rise to heavy tails in the distribution of stock price variations. Their model provides a link between the heavy tails observed in the distribution of stock market returns and herding behavior in financial markets. On the other hand, heterogeneous agents including deceivers and conservatives also are used to characterize the emerging collective behavior (Da Silva et al., 2006). Besides the interaction among traders, the collective behavior is also generated by interaction among stocks (Gopikrishnan et al., 2001; Pan and Sinha, 2007).

This paper proposes an explanation for option volatility smile based on the widespread collective behavior among stock market traders. We first employ an agent-based model to form the pricing process of underlying stock from the perspective of individual interaction. These price sequences of the underlying stock will be used to calculate option price through the Monte Carlo method. And then the implied volatility is calculated by the inverse of the Black and Scholes option price formula. The difference between our approach and others is the endogenous stock price sequence generated by an agent-based model with collective behavior, rather than the price process given exogenously, such as the stochastic volatility model and jump-diffusion model. The ideological framework for this paper is demonstrated in Fig. 1.

In our agent-based model, when the previous logarithmic return which means the price change is more dramatic, the stock traders behave more consistently and collectively, thus the number of the crowds/groups in the market endogenously becomes less. The equilibrium prices are generated through the trades among these collective traders in a continuous double auction market. The collective feature of our model makes option prices defined on the equilibrium price consistent with the implied volatility smile. Moreover, the effect of collective behavior is analyzed by a change of degree of collective behavior in our model. We will observe that the larger the degree of collective behavior, the more convex is the smile curve.

This paper is organized as follows. In Section 2, we describe the details of our agent-based model and how stock equilibrium price, option price and implied volatility are separately generated. In Section 3, we first characterize the statistical properties of the underlying asset equilibrium price. Next, we display the option volatility smile based on this equilibrium price. In addition, we give a comparable analysis to show how the degree of collective behavior can influence the shape of volatility smiles. Finally, Section 4 summarizes this paper.

2. The model

2.1. Market structure

In our agent-based model, the price is formed by continuous double auction. The continuous double auction is the most popular method of price formation in real financial markets. Traders can submit both limit orders and market orders to buy and sell. An order that does not cross the opposite quoted price and so does not lead to an immediate execution is called a limit order. An example is a buy order with a lower price than any existing sell order. An order that does cross the opposite quoted price and thus results in an immediate execution is called a market order. Limit orders accumulate in the order book, while market orders lead to trading that expend limit orders. A limit order can also be removed from the order book by cancelation. The lowest selling price is called the best ask price, $A(t)$, and the highest buying price is called the best bid price, $B(t)$. The equilibrium price below we used is defined as $P(t) = (A(t) + B(t))/2$. Orders are listed in the order book by price priority at the time level and time priority at the price level until they are executed or canceled. Fig. 2 shows the order book of continuous double auction.

2.2. Zero intelligence case

Zero intelligence traders have no strategy and can be divided into two types as in Farmer et al. (2005). Impatient agents place market orders randomly with the probability of p_a per unit time. Patient agent place limit orders randomly with the probability of p_b per unit time. The selections of buy or sell are random with the same probability. For the simplicity, all traders only submit one share in every trade. Based on these assumptions, a trading decision $\varphi_i^k(t)$ is made by each agent i,

$$\varphi_i^k(t) = \begin{cases} 1, & \text{with probability } p_k/2 \rightarrow \quad \textit{buy} \\ -1, & \text{with probability } p_k/2 \rightarrow \quad \textit{sell} \\ 0, & \text{with probability } 1-p_k \rightarrow \quad \textit{hold} \end{cases} \tag{1}$$

where k represents a or b. Patient agents, in addition, also need to decide the order price that will be placed. Buy limit orders are placed uniformly