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Analyses of retirement benefits with options

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ABSTRACT

This study applies the contingent claim approach to evaluate retirement benefits with the options of choosing the maximum defined benefit and defined contribution pension plans. A least-squares Monte Carlo simulation values complex retirement benefits that feature the properties of multiple variables, early exercise, stochastic interest rates, and several embedded options. Furthermore, this study examines the impacts of different forms of early decrements of the value of retirement benefits with options.

1. Introduction

The world is facing increasingly salient issues of aging as the population of people aged older than 65 years continues to grow.[1] In this context, retirement programs are becoming more important, because people must rely on the pension benefits to support themselves for the rest of their lives, after they retire from work. Retirement benefits are usually paid according to two approaches: defined benefits (DBs) or defined contributions (DCs). In Australia, Italy, and some states in the United States, retirement benefit programs are designed to offer the maximum of two alternative benefit values (DB and DC), equivalent to a better-of option (Smithson, 1998). Employees have the option to choose from the maximum of the two underlying assets when they retire (Johnson, 1987; Stulz, 1982). When an employee is allowed to retire early, American options also are embedded in the retirement benefit program.

Option pricing methodologies first were used in the valuation of investment guarantees by Boyle and Schwartz (1977). Wilkie (1989) discusses option pricing techniques for pension benefit payments in the United Kingdom. Shimko (1989, 1992) develops a contingent claim approach to value insurance claims. Sherris (1995) extends the contingent claim approach to the case of retirement benefits.

Ever since Black and Scholes (1973) conducted a renowned European option closed-form solution, the development and application of option evaluations have attracted concern from academia and practice. There also has been plenty of research and actual costs devoted to attempting to find a reasonable price of a derivative by the most efficient and precise method, to benefit both the exchange proceeding that of financial merchandises and analyzing strategies to manage risk. Except for the simplest European option, there are many other exotic options traded in the market. The Black and Scholes (1973) formula can only evaluate a simple European option, whereas several numerical methods exist to evaluate complicated exotic options.

The use of numerical techniques for valuing options is discussed in Hull (2006). Three types of numerical techniques have been frequently applied for option valuation: (1) approximate the underlying stochastic process directly by Monte Carlo simulation as first introduced by Boyle (1977); (2) use various lattice (tree) approaches, such as Cox et al.'s (1979) binomial tree method; or (3) discretize a partial differential

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[1] The United Nations (2009) has shown that the global population is aging and that the trend is persistent. The proportion of older persons (60 years or older) has been rising steadily from 8% in 1950 to 11% in 2009 and expected to reach 22% in 2050. Among those aged 60 years or older, the fastest growing population is the oldest-old (80 years or older), which is growing at a rate of 4% per year. Persons aged 80 years or older accounted for nearly 1 in every 7 older person in 2009. By 2050, this ratio is expected to increase to nearly 1 in every 5 older persons. These trends affect the potential support ratio (PSR), which refers to the number of persons aged 15 to 64 years for each person aged 65 years or older; it indicates how many potential workers there are per older person. As a population ages, the PSR tends to fall. Between 1950 and 2009, global PSR declined from 12 to 9 potential workers per older person (United Nations, 2009). By 2050, it is likely to drop further to 4 potential workers per older person. This reduction has important implications for social security schemes and pension systems.

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equation by finite difference methods, such as in Brennan and Schwartz (1978). Both the lattice model and finite difference method will be inefficient when the state variable number is large enough, because the memory space and computation time will grow exponentially as the state variable increases. Generally, lattice and finite difference methods become incapable of valuing options when the number of state variables increases beyond three.

On the contrary, the simulation approach seems to be the best choice for pricing complex options. The most important concern for numerical methods is their efficiency and convergence. Because the DC depends on the history of the salary growth rate, it is not a simple function of the final value of the state variables. Considering the path dependency and the multi-variable problem in the pension pricing, the lattice and finite difference techniques are impractical. It is more efficient to deal with the valuation of retirement benefits by simulation.

Boyle (1977) first proposed a simulation approach for pricing options. The simulation approach is very flexible and can be used to price complex European-style options. Before 1993, there were few published works on the use of simulation to value American options. Tilley (1993) was the first to develop such a technique, which included a procedure for incorporating an early exercise principle in the simulation. Tilley in his study mentions but does not demonstrate that it is possible to handle the pricing of American options with two or more state variables. Barraquand and Martineau (1995) subsequently proposed an approach that could track the conditional probabilities of path-specific outcomes in simulation. They used these values to make early-exercise decisions and value American put options.

Broadie and Detemple (1996) also developed a simulation algorithm that produces two estimators for the true option value, one biased high and the other biased low, and both asymptotically unbiased as the number of simulations tends to infinity. By the convergence principle, these two estimates provide a conservative confidence interval for the option values. Grant et al. (1996) demonstrated how to determine the critical prices before maturity at the first step by backward induction in simulation. Knowing the critical prices at each possible early exercise time before maturity, these authors illustrated in a second step the pricing of put options by forward induction using simulations. Their model is able to incorporate the early-exercise feature into simulations, and they showed that their approach could accurately value plain vanilla American options, but they did not reveal whether their approach could be applied to pricing American options depending on multiple assets. Raymar and Zwecher (1997) extended Barraquand and Martineau's (1995) approach by increasing the factors that separate the simulation paths from one to two. Their approach can effectively eliminate the pricing error that Barraquand and Martineau (1995) committed and thus increase the accuracy of simulation for pricing American options.

Longstaff and Schwartz (2001) developed a simple least-squares Monte Carlo simulation (hereafter LSM) method to price American options. LSM can easily handle complex option pricing problems with several variables and path-dependent exotic features. We apply LSM to solve the valuation problem of pension benefits in this article.

Some additional issues have to be considered when applying option pricing methods in pension valuation, such as the consideration of decrements of resignation, death, and disability into the calculation, as well as the path-dependent form of the benefit payment. This article extends the study of Sherris (1995) by relaxing some impractical assumptions that can fit reality well. It is unfeasible and will cause serious biases by assuming a fixed interest rate environment for the valuation of pension benefit with maturity of more than ten years. One of the key extensions is that we assume that the risk-free interest rate follows the Ito process, which Sherris (1995) assumes is deterministic.

The DC depends on the history of the salary growth rate, which means that DC is not a simple function of the final value of the state variables. Moreover, since an employee is allowed to retire earlier, the embedded option is a path-dependent American-style option. To build a practical model, we propose a LSM simulation approach, which can efficiently analyze retirement benefits with the properties of multi-variables, early exercise, and several embedded options. Series of sensitivity analyses of parameters to the value of retirement benefits and early retirement probabilities are presented. People can apply our model to evaluate the values of retirement benefits with options under different scenarios to make optimal retirement decisions.

The remainder of this article is organized as follows. Section 2 constructs the model. The state variable processes, pension systems and least squares Monte Carlo simulation applied to retirement are introduced in this section. Section 3 presents the numerical analyses for various scenarios. The impacts of stochastic interest rate and state variable correlations to pension benefits will be discussed. We finally draw conclusions and discuss implications of our findings in Section 4.

2. The model

2.1. State variable processes and pension benefits

The better-of option is assumed to be a function of three state variables: the crediting rate, the salary growth rate, and the interest rate. In this paper the crediting rate is the earning rate of a diversified portfolio that represents the asset portfolio of a typical retirement fund (Sherris, 1995). The salary growth rate (denoted by s), the crediting rate (denoted by f), and the risk-free interest rate (denoted by r) are assumed to follow the stochastic Ito processes as Eqs. (1)–(3) describe:

$$ds = \mu_s(s,t) + \sigma_s(s,t)dZ_s, \tag{1}$$

$$df = \mu_f(f,t) + \sigma_f(f,t)dZ_f, \text{ and} \tag{2}$$

$$dr = \mu_r(r,t) + \sigma_r(r,t)dZ_r, \tag{3}$$

where $dZ_i dZ_j = \rho_{ij}dt$; i, j = s,f,r; i ≠ j. The term ρ_{ij} denotes the instantaneous correlation coefficient between the standardized Wiener increments dZ_i and dZ_j.

For an employee who joins a pension program at time t at age x years and retires at time T, when he is (x + T − t) years old, the value of the benefits will be the maximum of the defined benefits D(x,t,T) and the defined contribution A(x,t,T). Here, D(x,t,T) denotes a fixed multiple of the final salary, and A(x,t,T) denotes the accumulated value as a fixed percentage of salary that grows at the crediting rate during the membership time of T − t. The ultimate age of membership in the retirement fund is assumed to be 65 years, which will occur at time t + 65 − x. The conditions for the employee who has the right to decide whether to retire early are assumed to be as follows: (1) work experience of more than 15 years and age is 55 years or above or (2) work experience of more than 25 years.

We assume that the decrements of death and disability are paid the greater of the benefits, but the decrement of resignation is paid based on the defined contribution, which is assumed to be the accumulation of a fixed percentage of the salary (Sherris, 1995). The defined benefit is a fixed multiple of the final salary, D(x,t,T) = k(T − t)S(x,t,T), where T − t is the membership in years, k is the salary multiple for the value of the benefit at exit, and S(x,t,T) is the salary at time T.

The membership of years of the defined benefit in practice is based on the final average salary, which is often based on the last three years' average. Without lost of generality and to simplify the processes, we just use the salary at exit as a proxy. The salary at exit is a function of the salary growth rate from the age at entry to the age at exit, and the process of the salary will be

$$dS(x,t,T) = s(T)S(x,t,T)dT, \tag{4}$$

where s(T) is the salary growth rate at time T.

The cash flows of dA(x,t,T) are similar to a notional security that has a negative continuous dividend equal to the contribution rate times the