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Exploring Stackelberg profit ordering under asymmetric product differentiation☆

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ABSTRACT

The literature on differentiated products only considers symmetric cross-price effects and shows that the profit ordering of firms in a sequential set-up is uni-directional. This paper shows that uni-directional profit ordering breaks down under asymmetric product differentiation. Above a unique cross-price effect level the follower's profit exceeds that of the leader. The reverse is true below this level. This result holds for both substitutes and complements.

1. Introduction

The existing literature shows that the profit ordering of firms in a sequential game is uni-directional when the firms' products are either substitutes or complements to one another. Specifically, the leader's profit always exceeds that of the follower if the products are substitutes and the reverse is true for the complements. A key underlying assumption behind this result is that the cross-price effects are symmetric or equal in magnitude. For example, Gal-Or (1985) shows that the leader's profit is higher (lower) than that of the follower under quantity (price) competition. Boyer and Moreaux (1987b) find that the leader's profit exceeds that of the follower when the cross-price impacts are symmetric in the case of substitutes. For homogeneous goods with concave demand and convex cost structures, Dastidar (2004) shows that the profits of the competing firms are equal under price competition. The leader's profit is higher under quantity competition.

However, several empirical studies suggest that product differentiation may not be symmetric which means that the cross-price effects can be asymmetric.[1] For example, in the context of the US automobile industry, Berry et al. (1995) show that the cross-price semi-elasticity between the Nissan Sentra and its substitute the Ford Escort is 1.375 while that between the Ford Escort and the Nissan Sentra is 8.024. In the case of the processed cheese market in the US, Kim and Cotterill (2008) find that the cross-price elasticity between the substitutes Kraft and Weight Watchers is 0.04 while that between Weight Watchers and Kraft is 0.25. Rojas and Peterson (2008) show that, in the US beer market, the cross-price elasticity between Bud and Old Style is 0.003 and that between Old Style and Bud is 0.242. From a theoretical perspective, Bonfrer et al. (2006) and Diewert (1980) show that aggregate demands need not satisfy any symmetry condition and also, at the individual level, the Slutsky matrix may not be symmetric because the income effect may not be symmetric. In the case of complements for example, operating systems are "better" complements to media software as changes in the price of media software does not impact the demand for operating systems as much as the price changes of operating systems affect the demand for media software.

Building on the observation that cross-price effects can be asymmetric, we move away from a one-dimensional to a two-dimensional concept of product differentiation and re-examine the question of profit ordering in a sequential game. A vertically differentiated product market is one example where the goods are asymmetric substitutes in the sense that the inferior product is an imperfect (and always a poorer) substitute of

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[1] Asymmetric (symmetric) product differentiation implies that the cross-price effects between two products are unequal (equal) in magnitude.

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the superior product hence, the cross price effects are asymmetric and they differ in magnitude. However, there is no reversal in the asymmetry and one product is always better than the other. Alternatively, one can also think of a horizontally differentiated product market where the consumers' preferences are such that it results in asymmetric product differentiation.[2] However, in both these settings the literature focuses on price competition and only considers substitutes but not complements.

While we consider both price and quantity competitions with asymmetric cross price effects, the objective of our paper is not to look at price or quantity competition separately but to compare the leader's and the follower's profits under these market competitions. We do this by considering both substitutes and complements in the analysis. The two-dimensional framework resulting from the asymmetric cross price effects generate the two dimensional concept of product differential that leads to some novel non-monotonic results, which is the main contribution of this paper.[3]

Most importantly, we show that the relative degree of complementarity or substitutability between the products determines the profit ordering of the firms. There exists a *unique critical* level of product differentiation, which creates a reversal in the profit ordering between the leader and the follower. Thus, in contrast to the finding of the literature on symmetric product differentiation, profit ordering ceases to be uni-directional with the introduction of asymmetric product differentiation.

The reversal of the profit ordering result can be intuitively explained as follows. Let us consider the case where the products are complements and suppose that the negative impact of the follower's price on the leader's quantity is *larger* than that of the leader's price on the follower's quantity. Consequently, the follower can cater to a higher market demand at a higher price compared to that of the leader. Hence, the follower's profit exceeds that of the leader. As the negative impact of the follower's price on the leader's quantity decreases the follower's profit also decreases and beyond a critical level of the cross-price impact the profit ordering reverses with the leader's profit exceeding that of the follower.

In the case of substitutes suppose the positive impact of the follower's price on the leader's quantity is *smaller* than that of the leader's price on the follower's quantity. Thus the follower can cater to a higher market demand at a higher price compared to the leader resulting in a larger profit for the follower. As the positive impact of the follower's price on the leader's quantity increases, the follower's profit decreases and beyond a critical level of the cross-price impact the profit ordering switches with the leader's profit exceeding that of the follower.

This paper is arranged as follows. In Section 2 we present the model and the basic results. Sections 3 and 4 contain the comparative static analysis and concluding remarks.

2. The model

Gal-Or (1985)'s seminal paper assumes that the two players in a Stackelberg game are identical and shows that the leader's profit is higher (lower) than that of the follower under quantity (price) competition. However, in our paper, we deviate from the assumption that the players are identical by assuming that the cross-price effects are asymmetric and hence the payoffs are different for the same strategy choices. We extend Gal-Or (1985)'s model by incorporating the asymmetric demand condition to demonstrate the reversal of profit ordering between the leader and the follower for both price and quantity competition.[4]

We consider two players i and j where $i, j = L,F, j \neq i$ with L and F denoting the leader and the follower in a sequential game. A player i's strategic choice is S_i which can be either price (P_i) or quantity (q_i). Let $\pi^i(s_i,s_j)$ denote player i's payoff. We assume that π^i is twice continuously differentiable, the cross partials exist, $\pi^i(s_i,s_j)$ is strictly concave in s_i, and $\pi^i(s_i,s_j)$ and $\pi^i(s_i,s_j)$ are strictly monotonic functions of S_j.

Like Gal-Or (1985), we assume that the payoffs are unaffected by the timing or order of the strategies. However, unlike Gal-Or (1985), we assume that the payoffs of the players are not the same for any pair of strategies. So, $\pi^i(v,w) \neq \pi^i(w,v) \neq \pi^j(w,v) \neq \pi^j(v,w)$ for every v and w. L's strategy is to choose $s_L \in [\underline{s}, \overline{s}]$ while F chooses a decision rule $s_F : [\underline{s}, \overline{s}] \rightarrow [\underline{s}, \overline{s}]$. The Nash equilibrium is defined as follows.

Definition 1. *The strategy choices* (s_L^*, s_F^*) *will be a Nash equilibrium in a sequential move game if* $s_F^* \equiv g(s_L^*) = \underset{s}{argmax}\, \pi^F(s_L^*, s)$ *and* $s_L^* = \arg\max_s \pi^L(s, g(s))$ *where* $g(.)$ *is F's reaction function*. [p649, Gal-Or (1985)].

Let us assume that a unique interior equilibrium exists. The implication of L's profit maximising objective subject to F's reaction function, is that, when the reaction function is positively sloped or $\pi^F_{LF}(s_F^*, s_L^*) > 0$, then the relation between s_L^* and s_F^* is indeterminate; that is, either $s_L^* < s_F^*$, or $s_L^* < s_F^*$ or $s_L^* < s_F^*$. Similarly, when the reaction function is negatively sloped or $\pi^F_{LF}(s_F^*, s_L^*) < 0$, then it can be that either $s_L^* < g(s_F^*)$ or $s_L^* < g(s_F^*)$ or $s_L^* < g(s_F^*)$. The proof is provided in the Appendix.

Hence, we cannot conclusively say that $\pi^F(s_F^*, s_L^*) > \pi^L(s_L^*, s_F^*)$ when S_L and S_F are strategic complements or the reaction function is positively sloped. Neither can we conclude that $\pi^F(s_F^*, s_L^*) > \pi^L(s_L^*, s_F^*)$ when S_L and S_F are strategic substitutes or the reaction function is negatively sloped. Therefore, the conclusions drawn by Gal-Or (1985) need not always hold. These results are summarized in Proposition 1.

Proposition 1.

(i) *When s_F and s_L are strategic complements with F's reaction function being upward sloping, then*: (*a*) *F's payoff exceeds that of L if* $s_L^* > s_F^*$ *and* $g(s_F^*) > s_L^*$; (*b*) *L's payoff exceeds that of F if* $s_L^* < s_F^*$ *and* $g(s_F^*) < s_L^*$; (*c*) *since the payoffs are twice continuously differentiable and continuous in the strategy space, there will be a unique case when* $\pi^F(s_F^*, s_L^*) = \pi^L(s_L^*, s_F^*)$ *in the space* $s_L^* > s_F^*$ *and* $g(s_F^*) < s_L^*$.

(ii) *When s_L and s_F are strategic substitutes with F's reaction function being downwards sloping, then*: (*a*) *L's payoff exceeds that of F if* $s_L^* < s_F^*$ and $g(s_F^*) < s_L^*$; (*b*) *F's payoff exceeds that of L if* $s_L^* > s_F^*$ *and* $g(s_F^*) > s_L^*$; (*c*) *since the payoffs are twice continuously differentiable and continuous in the strategy space, there will be a unique case when* $\pi^F(s_F^*, s_L^*) = \pi^L(s_L^*, s_F^*)$ *in the space* $s_L^* > s_F^*$ *and* $g(s_F^*) < s_L^*$.

Proposition 1 implies that it is not always the case that L enjoys the first mover's advantage when the reaction curves are downward sloping. Similarly F does not always enjoy the second mover's advantage when the reaction curves are positively sloping. In both cases of S_L and S_F being strategic complements or substitutes, there are three possibilities. F's payoff is either strictly higher or lower than that of L, or there exists a unique path where their payoffs are equal.

In the next section we demonstrate this using an example with linear demand curves when the demands faced by the two firms are not identical and the cross price effects are asymmetric.

[2] Consider a Hotelling duopoly model in a [0,1] interval where firm A is located at 1 and firm B is located at ½. In contrast to the standard setting assume that a consumer does not buy from any firm located to its left. Then B's potential measure of consumers is ½ and that of A is 1. This can also be a possible setup where product differentiation is asymmetric. We thank one of the referees for this example.

[3] That the cross price effects can be asymmetric and the profit ordering of the leader and the follower can be reversed has been demonstrated in the case of spatial competition with price leadership by Anderson (1987).

[4] Wherever possible, we strictly adhere to Gal-Or (1985)'s notations to facilitate comparability.