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On the optimal choices of R&D risk in a market with network externalities

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ABSTRACT

This paper develops the Hotelling spatial model investigating the optimal choices of R&D risk in a market exhibiting network externalities. Assuming that firms perform R&D projects with identical expected outcomes but different risk degrees, it is found that, under certain conditions, the level of R&D risk is higher in the presence of positive network externalities than otherwise under private optimum. Moreover, the private optimum is insufficient from the viewpoint of social welfare and the intensity of network externalities is important in determining the extent of inefficiency. However, the subsidies for R&D can change this inefficiency.

1. Introduction

The high uncertainty is an important feature of technological innovation (Arrow, 1962). When a firm executes a R&D program, it generally does not know the exact outcomes. Several scholars have analyzed the choices of optimal R&D efforts under uncertainty (see Boone, 2001; Cerquera, 2006; Milstein and Tishler, 2006; Tishler and Milstein, 2009). Moreover, a firm needs to estimate the risk degrees if the final results of R&D are uncertain. The following literature has studied the optimal choices of R&D risk (see Choi, 1993; Klette and de Meza 1986; Tishler, 2008; Wang et al., 2013; Xing, 2011).

Network externalities arise when the utility that a consumer obtains from a product or a service increases with the number of consumers of same products or services (Katz and Shapiro, 1985). Several markets present positive network externalities, for instance, operating systems, telecommunications, consumer electronics, etc. Katz and Shapiro (1994) suggested that a firm's innovation incentives may be altered by considerations of network effect. Some theoretical R&D work has considered network externalities (see Boivin and Vencatachellum, 2002; Kristiansen and Thum, 1997; Sääskilahti, 2006; Xing and Wang, 2008; Xing et al., 2009). However, they focus on how the network externalities influence the optimal choices of expenditure on R&D efforts. This paper investigates the impact of network externalities on the choices of the optimal R&D risk. In our framework, the aim of R&D is to increase the consumer reservation price.

The remainder of the paper is structured as follows. Section 2 describes the model setup and solves the private and social optimum. Section 3 provides a situation with corner solution. Section 4 concludes the paper.

2. The model

The basic model is a variant of Hotelling's spatial duopoly model (Hotelling, 1929). Consider a linear market denoted by [0,1], along which consumers of mass 1 are uniformly distributed according to their preferences for product. Two firms denoted by firm 1 and firm 2 supply products for consumers in the market. Each firm is allowed to locate at one endpoint of the interval. Suppose firm 1 locates at 0 and firm 2 locates at 1. The firms carry out R&D investment in order to improve consumer reservation price for their product.

A consumer buying from firm i ($i = 1, 2$) derives two parts utility. One part is the reservation price, denoted by $a_i = a + \lambda_i$, where a is the reservation price before R&D and λ_i is the improvement of reservation price from firm i's R&D efforts. The other part is the network utility depending on network externalities, denoted by αd_i, where $\alpha \geq 0$ measures the intensity of network externalities (note that the network externalities do not appear when $\alpha = 0$) and d_i is firm i's network size or install base expected by consumers (Kristiansen and Thum, 1997). The net utility of the consumer located at x is given by:

$$\begin{cases} u_1 = (a+\lambda_1)+\alpha d_1-p_1-tx, & \text{if bought from firm } 1, \\ u_2 = (a+\lambda_2)+\alpha d_2-p_2-t(1-x), & \text{if bought from firm } 2 \end{cases} \tag{1}$$

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where p_i denotes firm i's product price, tx and $t(1-x)$ represent the disutility caused by using a product not consisting with his/her own preference, and $t > 0$ stands for the degree of differentiation. We confine our attention to the case where the market is completely covered. Moreover, suppose each consumer purchases exactly one unit of the product form one firm. The marginal consumer indifferent between purchasing from either firm is located at $\bar{x}$, which is given by $u_1 = u_2$, i.e.

$$(a+\lambda_1)+\alpha d_1-p_1-t\bar{x}=(a+\lambda_2)+\alpha d_2-p_2-t(1-\bar{x}). \tag{2}$$

Similar to Katz and Shapiro (1985), the consumers' expectations on firm's network size are fulfilled, i.e. d_i is equal to the actual demand of firm i ($i = 1, 2$). All consumers located to the left (resp. the right) of the marginal consumer will buy from firm 1 (resp. firm 2). Thus, d_1 and d_2 meet $d_1=\int_0^{\bar{x}}dx=\bar{x}$ and $d_2=\int_{\bar{x}}^{1}dx=1-\bar{x}$. Solving Eq. (2), we have:

$$\bar{x}=\frac{t-\alpha+\lambda_1-\lambda_2+p_2-p_1}{2(t-\alpha)}. \tag{3}$$

Suppose that both firms' marginal costs are equal to zero, and that their fixed costs are only caused by R&D investment. The firm i's profit function is given by:

$$\pi_i=p_id_i-I(\mu_i,\sigma_i),\quad i=1,2 \tag{4}$$

where $I(\mu_i,\sigma_i)$ denotes firm i's investment cost. Suppose the R&D outcomes for firm i (i.e. λ_i) are uncertain when it engages in product innovation. The probability distribution of λ_i is $\lambda_i \sim [\mu_i, \sigma_i]$, in which $\mu_i \in [0, \infty)$ and $\sigma_i \in [0, \infty)$ denote the expected value and variance respectively (i.e. $E(\lambda_i) = \mu_i$ and $V(\lambda_i) = \sigma_i$). Notice that the variance of R&D outcome measures the risk of R&D program and the firms are assumed to be risk neutral in this study. The R&D investment cost function for firm i has the following structure:

$$I(\mu_i,\sigma_i)=f(\mu_i)+g(\sigma_i),\quad i=1,2. \tag{5}$$

Suppose that $f' \geq 0$, $g' \geq 0$, and $g'(0) = 0$, and that $g'' > 0$ ($\sigma_i \neq 0$), which ensures the second-order conditions for R&D are met and the optimal solutions of R&D risk are interior.

The game runs as follows. In the first stage, two firms simultaneously choose the risk of their R&D programs. In the second stage, the firms simultaneously decide their product price.

2.1. The private optimum

As usual, the equilibrium results are solved by backwards induction.

2.1.1. Stage 2: each firm decides price

In this stage, both firms set product price so as to maximize their profit function, taking their rival's product price and the outcomes of R&D program as given. Solving the first-order conditions, we obtain equilibrium prices:

$$p_1^e=\frac{3(t-\alpha)+(\lambda_1-\lambda_2)}{3} \tag{6}$$

$$p_2^e=\frac{3(t-\alpha)-(\lambda_1-\lambda_2)}{3}. \tag{7}$$

When $t > \alpha$, the second-order conditions ($\frac{\partial^2\pi_i}{\partial p_i^2}=-\frac{1}{t-\alpha}<0, i=1,2$) are satisfied. Suppose the intensity of network externalities meets the above inequality in this paper. The resulting demands and profits are respectively given by:

$$d_1=\frac{3(t-\alpha)+(\lambda_1-\lambda_2)}{6(t-\alpha)} \tag{8}$$

$$d_2=\frac{3(t-\alpha)-(\lambda_1-\lambda_2)}{6(t-\alpha)} \tag{9}$$

$$\pi_1=\frac{[3(t-\alpha)+(\lambda_1-\lambda_2)]^2}{18(t-\alpha)}-I(\mu_1,\sigma_1) \tag{10}$$

$$\pi_2=\frac{[3(t-\alpha)-(\lambda_1-\lambda_2)]^2}{18(t-\alpha)}-I(\mu_2,\sigma_2). \tag{11}$$

Note that, in order to ensure that both firms' demand is positive, the ex post reservation price difference between the firms has to satisfy $|a_1 - a_2| = |\lambda_1 - \lambda_2| < 3(t - \alpha)$.

2.1.2. Stage 1: each firm opts for R&D risk

Now the firms simultaneously choose the risk of their R&D program. Since the R&D outcomes are uncertain in this stage, firms determine the optimal R&D choices to maximize their expected profit. The expectation for Eqs. (10) and (11) are:

$$E(\pi_1)=\frac{[3(t-\alpha)+(\mu_1-\mu_2)]^2+\sigma_1+\sigma_2-2\,\mathrm{cov}(\lambda_1,\lambda_2)}{18(t-\alpha)}-I(\mu_1,\sigma_1) \tag{12}$$

$$E(\pi_2)=\frac{[3(t-\alpha)-(\mu_1-\mu_2)]^2+\sigma_1+\sigma_2-2\,\mathrm{cov}(\lambda_1,\lambda_2)}{18(t-\alpha)}-I(\mu_2,\sigma_2) \tag{13}$$

where $\mathrm{cov}(\lambda_1, \lambda_2)$ denotes the covariance of λ_1 and λ_2, and is assumed as a constant.

The firms have to evaluate the risk when performing R&D programs because the uncertainty exists. Similar to Tishler (2008), this study focuses on the optimal choices of R&D risk by comparing R&D programs with identical expected outcomes (i.e. $\mu_1 = \mu_2$). The first-order conditions of Eqs. (12) and (13) lead to:

$$\frac{1}{18(t-\alpha)}-\frac{\partial I(\mu_1,\sigma_1)}{\partial\sigma_1}=0 \tag{14}$$

$$\frac{1}{18(t-\alpha)}-\frac{\partial I(\mu_2,\sigma_2)}{\partial\sigma_2}=0. \tag{15}$$

Substituting Eq. (5) into Eqs. (14) and (15), we obtain:

$$\frac{1}{18(t-\alpha)}-g'(\sigma_i^e)=0,\quad i=1,2 \tag{16}$$

where σ_i^e is firm i's equilibrium risk level. According to Eq. (16), we can prove that $\sigma_1^e = \sigma_2^e$. Let $\sigma_i^e = \sigma^e$ and $i = 1, 2$.

We set $\eta^e=\frac{1}{18(t-\alpha)}$ and then obtain $\eta^e|_{t>\alpha>0} > \eta^e|_{\alpha=0}$. Combining with Eq. (16), we derive the following proposition if $|a_1 - a_2| < 3(t - \alpha)$.

Proposition 1. *The equilibrium R&D risk level is higher in the presence of network externalities (i.e. $\alpha > 0$) than otherwise (i.e. $\alpha = 0$).*

This proposition implies that the risk-neutral firms will opt for more risky R&D programs when the positive network externalities appear rather than when they are absent. The intuition behind this is as follows. If the consumer reservation price for a firm's product is low, few users are attracted to buy its product as the network size is too small. Thus, when the network externalities present, the R&D investment has an extra effect on firms' profit through the increase in consumer reservation price which attracts new users and as a result improves the willingness to pay of all users (Boivin and Vencatachellum, 2002). That is, the network externalities can strengthen the impact of R&D on firms' profit. Moreover, according to Eqs. (12) and (13), the expected gross profit for firm i, $E(\Pi_i) = E(\pi_i) + I(\mu_i,\sigma_i)$ ($i = 1, 2$), increases as the risk of its R&D program increases (Tishler, 2008). Therefore, the