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# Relationship between the benchmark interest rate and a macroeconomic indicator

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ABSTRACT

A Poisson process with stochastic intensity is utilized to model changes of a benchmark interest rate set by a Central Bank. We propose explicit formulas for estimators of parameters and the expectation of the intensity, based on observations of the process. Through comparing the intensity and an economic indicator, we can explore the pattern of the benchmark interest rate. Two empirical datasets are studied and the results reveal similarities and differences between the behavior and the goals of Central Banks.


## 1. Introduction

The paper studies the relationship between a macroeconomic indicator and a benchmark interest rate set by a Central Bank. The benchmark interest rate is one of the most important tools by which a monetary policy maker carries out its monetary policy. Through exploring the relation between a benchmark interest rate and a macroeconomic indicator, we hope to discover the behavioral pattern and interests of a monetary policy maker.

The benchmark interest rates are different for Central Banks of countries. In America, a benchmark interest rate is the federal funds target rate (FFTR), which is set by the Federal Reserve Board. In the United Kingdom, a benchmark interest rate is the official bank rate, managed by the Bank of England. In China, the benchmark interest rate is determined by the People's Bank of China and the interest rate is called the benchmark deposit and lending interest rate. In most countries, benchmark interest rates and Central Banks are unambiguous. Among many different interest rates, a benchmark interest rate in a country has some features, which ensure that a point process is a good model for a benchmark interest rate.

By changing a benchmark interest rate, a Central Bank affects the money supply and macroeconomic outcomes of a country. There are various economic indicators of growth, inflation, employment, and so on, which describe aspects of the macroeconomy of a country. On the other hand, a change of a benchmark interest rate is a final decision after considering government policy, supply and demand, and macroeconomic outcomes of a country. We will explore the relationship between a macroeconomic indicator and a benchmark interest rate.

The remainder of the paper is organized as follows. In Section 2, a literature review is conducted to analyze existing models and algorithms. By investigating the strengths and limitations of these models, we select a point process as the model of a benchmark interest rate. Section 3 presents the methodology. The section includes the model of a benchmark interest rate, an explicit formula of the latent intensity process, and an algorithm estimating the similarity between the intensity and a given macroeconomic indicator. Section 4 presents two empirical studies for real data sets of interest rates. In both cases, we infer a relationship between a macroeconomic indicator and a benchmark interest rate. Section 5 concludes the paper.

## 2. Literature review and model selection

### 2.1. Literature review

There has been active research on interest rates for more than a hundred years. Although there is a long history of the theory of interest rates, relations between economic indicators and interest rates receive scant attention in much of the literature. Scattered literature analyzes relations between different economic indicators and interest rates. Through analyzing the relationship between interest rates and inflation, Carr (2011) pointed out that when the expected inflation increases,

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a nominal interest rate also increases. Carr's result agrees with the well-known Fisher effect. Many researchers have studied the Fisher effect and an explicit formula, the Taylor rule, was proposed to describe the phenomenon, see Thornton (1998), Taylor (1999), Bar-Ilan (2010), and references therein for details. Besides, Rigobon and Sack (2003, 2004) studied the relation between an interest rate and a stock price index. Rigobon and Sack pointed out that a 5% rise (fall) in the S&P 500 index corresponds to 70% chance of interest rate's tightening (easing). Further, Goukasian and Majbouri (2010) studied some economic indicators relevant to real estate-related industries. Goukasian pointed out that these indicators react strongly to changes of the federal funds target rate (FFTR). Recently, Chun (2011) studied the relation between GDP and interest rates. There is a long list of macroeconomic indicators whose relation with interest rates has been explored. The list includes inflation, stock price indexes, and GDP.

The literature offers us many statistical models and algorithms applicable to interest rates. We refer to Campbell and Shiller (1991) for a review of models and algorithms. As traditional models of interest rates cannot reflect some specific features of the real world, many researchers are concerned about nonstationary and nonlinear statistical models and algorithms. Carr (2011) used a curve fitting method to analyze a nonlinear regression relationship between inflation data and interest rates. Rigobon and Sack (2004) used heteroskedasticity-based estimators to estimate the response of interest rates to the changes of monetary policy. Rigobon and Sack (2003) used a vector autoregression model, and proposed an identification technique for interest rates. Second-generation nonlinear models of interest rates include wavelet artificial neural network analysis (WANN), mixed spectrum analysis (MS) and nonlinear ARMA models with Fourier coefficients (FNLARMA), see Nachane and Clavel (2008) and references therein for more details. By comparing simulation results, Nachane and Clavel found that WANN and MS have better performance than FNLARMA.

Recently, some researchers put forward point process models for a benchmark interest rate set by a Central Bank. Hamilton and Jordà (2002) proposed the autoregressive conditional hazard (ACH) model, and Grammig and Kehrle (2008) utilized marked point process models combining the autoregressive conditional multinomial (ACM) model to describe the irregular durations of benchmark interest rate changes. Dolado and María-Dolores (2002) also used a marked point process model to evaluate the target rate set by the Bank of Spain. A marked point process model puts a threshold for a latent stochastic process and assumes that interest rates do not change while the latent process is below the threshold. Then, considering the peculiarity of a benchmark interest rate, Gutiérrez and Lozano (2012) used competing risks survival analysis to model benchmark interest rates. Gutiérrez and Lozano assumed that duration of a benchmark interest rate has an exponential distribution but the mean of the duration varies according to exogenous indicators.

### 2.2. Model selection

There are two marked features of a benchmark interest rate. First, to ensure the stability of its monetary policy, a Central Bank does not change a benchmark interest rate frequently. Second, the magnitude of a benchmark interest rate's change is often one of several fixed amounts. For example, in the last twenty years, FFTR has been changed no more than fifty times, and all changes are multiples of 25 basis points. Therefore, many statistical methods and algorithms, which assume that interest rates are real value stochastic processes and vary continuously, cannot adapt to the benchmark interest rate set by a Central Bank. Due to the above features, a point process is a good model of a benchmark interest rate. A point process can describe the total number of "events" that occur during a time interval. In our case, the "events" is the change of a benchmark interest rate, and the time interval between two consecutive events is duration of a benchmark interest rate.

The Poisson process is an important and simple point process. However, a Poisson process does not appear in most of the literature of a marked point process model. For example, Dolado and María-Dolores (2002), Hamilton and Jordà (2002), Grammig and Kehrle (2008), and Gutiérrez and Lozano (2012) do not use a Poisson process as a model for a benchmark interest rate. This may be because a Poisson process possesses stationary increments, that is, duration of a benchmark interest rate has a constant mean, which does not resemble a benchmark interest rate.

Gutiérrez and Lozano (2012) assumed that duration of a benchmark interest rate has an exponential distribution with varied mean. We adopt the assumption of Gutiérrez and Lozano and utilize a Poisson process with stochastic intensity as a model for a benchmark interest rate. Moreover, by exploring the properties of a Poisson process with stochastic intensity, we can integrate changes of a benchmark interest rate with different magnitudes into the model. Gutiérrez and Lozano divided changes of FFTR into groups according to magnitudes and investigated each group separately. Comparing with Gutiérrez and Lozano (2012), from the same data, we may study more features of the benchmark interest rates in a unique framework.

## 3. Methodology

### 3.1. Model

We use a Poisson process $\{N(t); t \geq 0\}$ with stochastic intensity $\lambda(t)$ to model actions of a monetary policy maker. An orbit of such a process is nondecreasing right-continuous, $N(0) = 0$, and every increment $N(t + 0) - N(t)$ is 1 or 0. We define $\tau_0 = 0$ and let $\tau_i$ be the $i$-th jumping time, that is,

$$\tau_i = \inf\{t : N(t) = i\}, \quad i \geq 1.$$

As well known, we may describe the process $\{N(t); t \geq 0\}$ by the jumping times $\tau_0, \tau_1,$, which have probability density functions. In the case that $\lambda(t) = f(t)$, where $f(t)$ is a positive deterministic function, the probability density function (pdf) $g(u_1,\ldots,u_n)$ of $\tau_1,\ldots,\tau_n$ has the following form.

$$g(u_1, \ldots, u_n) = \begin{cases} \prod_{i=1}^{n} f(u_i) \cdot \exp\left(-\int_0^{u_n} f(s)ds\right), & 0 \leq u_1 < \ldots u_n, \\ 0, & \text{otherwise}. \end{cases} \quad (1)$$

For a stochastic intensity $\lambda(t)$, Eq. (1) provides the pdf of $\tau_1,\tau_2,\ldots,\tau_n$ conditional on $\lambda(t) = f(t)$.

If the intensity is a random variable, $\tau_i - \tau_{i-1}$ has a conditional exponential distribution, which is the model of Gutiérrez and Lozano (2012).

Although the formal notation of the model is clunky, the concept of the model is simple. Here $N(t)$ denotes the number of actions of a monetary policy maker until time $t$. Roughly speaking, the i-th action of the monetary policy maker happens at $\tau_i$. Here actions describe changes of interest rates.

In nature, there are many patterns of a monetary policy maker's behavior. For example, when changing a benchmark interest rate, a monetary policy maker may increase or decrease it. Besides, the magnitude of a change has a few different choices. We will reveal the reasonability of the model for all these cases.

Objectives of a monetary policy maker include the maximum employment, stable prices, and a moderate long-term interest rate, see Goukasian and Cialenco (2006) and Gerdesmeier et al. (2007) for more details. We can assume that there is a reference indicator determined by the monetary policy maker based on macroeconomics. Through adjusting the benchmark interest rate, the maker tries to get the reference indicator as close as possible to an ideal value. When the reference indicator is far beyond its ideal value, the maker may take