Economic Modelling 38 (2014) 258–269

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

Conditional Markov regime switching model applied to economic modelling

Stéphane Goutte *

Université Paris 8 (LED), 2 rue de la Liberté, 93526 Saint-Denis Cedex, France
Affiliated Professor, ESG Management School, 25 rue Saint-Ambroise 75011 Paris, France

ARTICLE INFO

Article history:
Accepted 19 December 2013
Available online 5 February 2014

JEL classification:
F31
C58
C51
C01

MSC classification:
91G70
60J05
91G30

Keywords:
Markov regime switching
Expectation–Maximization algorithm
Mean-reverting
Local volatility
Economic data

ABSTRACT

In this paper we discuss the calibration issues of regime switching models built on mean-reverting and local volatility processes combined with two Markov regime switching processes. In fact, the volatility structure of these models depends on a first exogenous Markov chain whereas the drift structure depends on a conditional Markov chain with respect to the first one. The structure is also assumed to be Markovian and both structure and regime are unobserved. Regarding this construction, we extend the classical Expectation–Maximization (EM) algorithm to be applied to our regime switching model. We apply it to economic data (Euro/Dollar (USD) foreign exchange rate and Brent oil price) to show that such modelling clearly identifies both mean reverting and volatility regime switches. Moreover, it allows us to make economic interpretations of this regime classification as in some financial crises or some economic policies.

© 2014 Elsevier B.V. All rights reserved.

1. Introduction

The use of Hamilton's Markov switching models to study economic time series data such as the business cycle, economic growth is not or unemployment new. In his seminal paper (Hamilton, 1989), Hamilton already noticed that Markov-switching models are able to reproduce the different phases of the business cycle and capture the cyclical behaviour of the U.S. GDP growth data. More recently, Bai and Wang (2011) went one step further by allowing for changes in variance and showed that their restricted model clearly identifies both short-run regime switches and long-run structure changes in the U.S. macroeconomic data. Janczura and Weron (2011) showed that Markov regime switching diffusion well fits market data such as electricity spot prices and allows useful economic interpretations of regime states. Goutte and Zou, (2013) compared the results given by the good fit of different regime switching models against nonregime switching diffusion on foreign exchange rate data. They proved that regime switching models with both mean reverting and local volatility structures are the best choice to fit data well. Moreover, this modelling allows them to capture some significant economic behaviour well, such as crisis time periods or change in the dynamic level of variance.

Based on the above observations that Markov switching models capture economic cycles and regime switching, we would like to extend the model stated by Goutte and Zou (2013) with a conditional Markov chain structure as in Bai and Wang (2011). Indeed, in Bai and Wang, (2011), the authors ignored the points that the model could have a mean reverting effect and a regime switching local volatility structure. As mentioned before, Goutte and Zou (2013) proved that a continuous time regime switching model better fits economic time series data than a non-regime switching model. Hence, in this paper, we will define a mean reverting local volatility regime switching model where the volatility structure will depend on a first Markov chain and the drift structure will have a mean reverting effect which depends on a conditional Markov chain with respect to the first one.

* Université Paris 8 (LED), 2 rue de la Liberté, 93526 Saint-Denis Cedex, France.

0264-9993/$ – see front matter © 2014 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.econmod.2013.12.007

Table 1
Summary statistics.

	Minimum	Maximum	Mean	Std. dev.	Skewness	Kurtosis
Data	0.832400	1.584900	1.218614	0.196548	−0.003278	0.003184

One of the objectives of this class of regime switching stochastic models is to capture various key features of the data such as mean trend gap or recession in a same economic level state of market volatility. In particular, in a possible high regime volatility state, our class of model will be able to capture different possible trends of the long mean average such as increasing or recession periods.

Thus, unlike Goutte and Zou (2013), our more general model will allow us to distinguish different possible economic states for the drift component for the same level of volatility.

We will develop an Expectation–Maximization (EM) algorithm to apply to this class of regime switching model. Indeed, the (EM) algorithm initiated by Hamilton (1989) is a two-step algorithm: firstly an estimation procedure in which we evaluate all the probabilities of the regime switching model; secondly a likelihood maximization step to estimate all the parameters of our models. Hence, in this paper, we will follow these two steps to give the procedure in our specific regime switching model case. Finally, since one of the aims of this paper is to establish a model that could capture various key features or trends of economic time series data, such as a mean level change or growth of volatility, we will use it on some economic time series data: the Euro/Dollar foreign exchange rate and the Brent crude oil spot price in Euros.

Hence, the paper is structured as follows: in a first section, we provide some notations and introduce our model. Then, in a second part, we set out the (EM) algorithm, which is the method for estimating all the parameters of our regime switching model. Then in the last section, we apply this method to economic time series data. We also give economic interpretations and so we show the ability of this regime switching model to capture various key features such as spikes in data or changes in the volatility level or crisis time periods.

2. The model

Let $T > 0$ be a fixed maturity time and denote by $(\Omega, \mathbb{F} := (\mathcal{F}_t)_{[0,T]}, \mathbb{P})$ an underlying probability space. In this paper, we will follow the seminal Markov switching model introduced by Hamilton (1989). However, in the sequel we will use a generalization of this classical regime switching model. First, we use a conditional Markov chain as initiated by Bai and Wang (2011). Secondly, we employ a more global class of stochastic model using a mean-reverting local volatility regime switching diffusion instead of a basic autoregressive model.

2.1. Conditional Markov chain

We begin with the construction of our Markov regime switching model. We will classify the states of the economy into exogenous and endogenous regimes characterizing long-run structure changes and short-run business cycles, respectively. The exogenous regime values will be given by a homogenous continuous time Markov chain X^2 on finite state $\mathcal{K} := \{1, 2,\ldots,K\}$ and with transition matrix P^{X_2} given by

$$P^{X_2} = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1K} \\ p_{21} & p_{22} & \cdots & p_{2K} \\ \vdots & \vdots & \vdots & \vdots \\ p_{K1} & p_{K2} & \cdots & p_{KK} \end{pmatrix}. \quad (1.1)$$

Remark 1.1. *The quantity p_{ij} represents the intensity of the jump from state i to state j.*

xThe endogenous regime values will be given also by a homogenous continuous time Markov chain X^1 on finite state $\mathcal{L} := \{1, 2,\ldots, L\}$ but its transition matrix will depend on the value of the exogenous regime. Hence, the transition matrix of X^1 will be conditional on the value of the Markov chain X^2. The endogenous economic regime thus follows a conditional Markov chain, where the Markovian property applies only after conditioning on the exogenous state. Hence, the state of the endogenous regime X^1 will be determined by conditioning on the state of the exogenous regime X^2.

To define the transition matrix of X^1 we first construct a time grid partition of the time interval $[0, T]$. For this, we partition the time interval such that,

$$0 = t_0 < t_1 < \cdots < t_N = T \quad \text{with} \quad \Delta_t := t_{k+1} - t_k = 1 \quad \text{for all} \quad k \in \{0,\ldots,N\}. \quad (1.2)$$

For all $s \in \mathcal{K}$, we can now define the probability transition to state $i \in \mathcal{L}$ to $j \in \mathcal{L}$ with respect to the value of the Markov chain X^2 of the Markov chain X^1 as

$$p^s_{ij} = \mathbb{P}\left(X^1_{t_k} = j \middle| X^2_{t_k} = s, X^1_{t_{k-1}} = i\right), \forall k \in \{1,\ldots,N\}, \forall s \in \mathcal{K}. \quad (1.3)$$

Hence we get K possible transition matrix $P^{X_1}_s$, $s \in \mathcal{K}$ given by

$$P^{X_1}_s = \begin{pmatrix} p^s_{11} & p^s_{12} & \cdots & p^s_{1L} \\ p^s_{21} & p^s_{22} & \cdots & p^s_{2L} \\ \vdots & \vdots & \vdots & \vdots \\ p^s_{L1} & p^s_{L2} & \cdots & p^s_{LL} \end{pmatrix}. \quad (1.4)$$

We assume in the what follows that

Assumption 1.1.

1. *For all $k \in \{1,\ldots, N\}$, $X^2_{t_k}$ is an exogenous Markov process. Hence, it satisfies*

$$\mathbb{P}\left(X^2_{t_{k+1}} | X^2_{t_k}, X^1_{t_k}, X^2_{t_{k-1}}, X^1_{t_{k-1}}, \ldots, X^2_{t_0}, X^1_{t_0}\right) = \mathbb{P}\left(X^2_{t_{k+1}} | X^2_{t_k}\right). \quad (1.5)$$

2. *For all $k \in \{1,\ldots, N\}$, $X^1_{t_k}$ is conditionally Markovian:*

$$\mathbb{P}\left(X^1_{t_{k+1}} | X^2_{t_{k+1}}, X^1_{t_k} X^2_{t_{k-1}}, X^1_{t_{k-1}}, \ldots, X^2_{t_0}, X^1_{t_0}\right) = \mathbb{P}\left(X^1_{t_{k+1}} | X^2_{t_{k+1}}, X^1_{t_k}\right). \quad (1.6)$$

Point 2 of Assumption 1.1 means that the value of the Markov chain X^1 at time t_k, $k \in \{1,\ldots, N\}$ depends both on the value of the Markov chain X^1 at time t_{k-1} and of the Markov chain X^2 at time t_{k-1}.

Remark 1.2. *In the particular case where $\mathcal{K} \equiv \mathcal{L} := \{1, 2\}$ and under Assumption 1.1, this model can be defined by the joint distribution $Z_{t_k} = \left(X^1_{t_k}, X^2_{t_k}\right)$ in the space $\mathcal{S} := \{(1, 1), (1, 2), (2, 1), (2, 2)\}$. Hence, in this two-regimes case, the transition matrix of the Markov chains X^1 and X^2 is given by:*

$$P^{X_2} = \begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix}$$

Table 2
Log likelihood value, AIC, BIC, RCM statistics and smoothed probability indicator given by the (EM) procedure for different values of δ.

δ	Log L	AIC	BIC	RCM(K = 4)	$P^{10\%}$	$P^{5\%}$
0	1128.0	−2287.9	−2158.0	33.15	70.70%	60.96%
0.5	1150.1	−2332.2	−2202.3	22.63	82.32%	78.25%
1	1172.4	−2376.8	−2246.8	38.53	65.36%	58.04%
1.5	1193.0	−2417.9	−2288.0	56.61	52.70%	42.40%

Download English Version:

https://daneshyari.com/en/article/5054374

Download Persian Version:

https://daneshyari.com/article/5054374

Daneshyari.com