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A recursive formula for a participating contract embedding a surrender option under regime-switching model with jump risks: Evidence from stock indices ☆

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ABSTRACT

This study proposes a recursive formula to value a surrenderable participating contract. To capture the dynamics of stock returns over expansion–recession cycles and the occurrence of catastrophic events, we assume the rate of return of the reference portfolio would follow a regime-switching model with jump risks. Our empirical results show that compared to the Black–Scholes model and the regime-switching model, the regime-switching model with jump risks can better explain the dynamics of the S&P 500 stock index. In addition, we give a recursive formula of a participating contract embedding a surrender option under a regime-switching model with jump risks. Sensitivity analysis shows that the changes of parameters of the regime-switching model with jump risks did influence participating contract premiums. The differences between valuations under the Black–Scholes model, the regime-switching model and the regime-switching model with jump risks suggest that it is critical to apply an appropriate model to value precisely a participating contract.

1. Introduction

In an era of low interest rates and high inflation, it is difficult for investors to accumulate real wealth by only depositing money in the bank. Smart investors put their money in different kinds of capital markets to earn higher return, such as the markets of: stock, bond, insurance, and mutual funds. However, each market has specific risks for investors. It is difficult to select an industry or company to invest in on the stock market as the market risk is high. Regarding the mutual fund market, it is necessary for the investor to discuss with a fund manager about portfolios regularly, for the investment may go back to the original value, depending on the business cycle, after fifteen to twenty years. In relation to the bond market, it is important to consider the credit ranking of the issuer. Compared to the other markets, risk in the insurance market is relatively small. Moreover, insurance products secure benefit when the insured dies, and are therefore good investment instruments.

In the past, insurance policyholders could only buy nonparticipating life insurance, which is cheaper than other insurance products incorporating investment, but only guarantees the benefit on the individual's death. Moreover, under this arrangement surrendering or switching the policy destroys the contract value when the interest rate goes up. Insurance subsequently evolved to incorporate investment, generally categorized as: universal life insurance, variable life insurance, and participating life insurance. Universal life insurance modifies the shortcoming of nonparticipating life insurance, by avoiding returns lower than the technical rate as the interest rate goes up. In fact, the value of universal life insurance is closely linked to the interest rate. More specifically, the value accumulates slowly when the interest rate goes down, and grows quickly when it goes up. The value of variable life insurance is linked to the performance of the portfolio allocated by the policy holder, who has to take full responsibility for profits or losses and therefore it is appropriate for aggressive investors.

Compared with the above mentioned insurance contracts, participating contracts have many advantages. Unlike universal life insurance, which gives only fixed interest income or variable life insurance that has possible investment loss, a participating policy is characterized as allowing policyholders to participate in the upside returns of the reference portfolio. Such a participating mechanism applies when "dividends" are credited to the policy reserve, thus increasing the insured's benefits. A participating contract with a minimum interest rate guarantee forces both the benefit and the periodical premiums to be adjusted annually according to the performance of a special investment portfolio. Moreover, the insured's benefit remains constant if the dividends part that insurance company wants to share with policyholders is lower than the minimum interest rate guarantee. By

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contrast, the benefit increases if the dividend is higher than the minimum interest rate guarantee and thus the payoff mechanism of these contracts is like European call options. Brennan and Schwartz (1976) pioneered the pricing of participating life insurance policy with an asset value guarantee under the Black–Scholes model. Further, Boyle and Schwartz (1977) valued a participating contract with both death and maturity benefit guarantees under this model. In addition, Grosen and Jørgensen (2000), Jensen et al. (2001) and Grosen and Jørgensen (2002) analyzed a participating policy, for which they used the Monte Carlo simulation to derive the percentage of positive performance of firm asset portfolios. Miltersen and Persson (2003) extended this to a multi-period contract, deriving closed-form formulae for pricing under a stochastic interest rate with the Heath–Jarrow–Morton (HJM) model. Bacinello (2001) used the Black–Scholes model to analyze life insurance endowment participating policies with a guaranteed minimum interest rate, and obtained closed-form formulae for those policies in terms of one-year call options.

A surrender mechanism is an American-style put option that entitles the policyholder to sell back the contract to the insurer at the cash surrender value. That is, as a participating contract embedding a surrender option it gives the policyholder the right to terminate the contract early at surrender value. To price a participating contract embedded with a surrender option it is necessary to consider three parts: valuations of the basic contract, participating option and surrender option. Albizzati and Geman (1994) took surrender options into account and derived a single-premium contract under the portfolio consisting of a zero coupon bond and stochastic interest rates. Grosen and Jørgensen (2000) and Jensen et al. (2001) priced surrender options embedded in participating policies with a binomial tree approach and a finite difference one, respectively. Bacinello (2003a) employed Cox et al. (1979) discrete option pricing model to derive a recursive formula to price: the basic contract, the participation option, and the surrender option.

As the pricing of both the participation and surrender options are affected by the value of the reference portfolio, it is important to identify its dynamics. During the past decades, in the high-yielding era, insurance companies were able to put most of their assets into bank deposits or bonds, with only a small portion having to be invested in high-risk assets, such as stocks or mutual funds, as they could still afford the minimum interest rate guarantee embedded in surrenderable participating contracts. However, recent near-zero interest rate policies implemented by governments worldwide have forced insurers to invest most of their assets in high-risk assets, such as stocks or mutual funds. Therefore, when pricing a surrenderable participating contract lasting around twenty years, it is critical to capture the dynamics of stock returns over expansion–recession cycles and the occurrence of catastrophic events.

Fig. 1 shows the dynamics of price and return of the S&P 500 index from 1999 to 2008 and it can be seen that stock prices were trending down from 2000 to 2003, whereas since 2003 the economy has recovered and share prices have been trending up. However, owing to the global financial crisis in 2008, share prices began trending downward again. Generally, the dynamics of price and return of the S&P 500 can be classified as an expansion–recession cycle, in which expansion represents stock price trending upwards, while recession represents it trending downwards. A similar idea was also introduced in Hamilton (1989), who stated that the economy is in expansion if the growth rate of GNP is positive, and the economy is in recession, if the growth rate of GNP is negative. Past research has shown that the regime-switching model can describe features in different market states (Alizadeh and Nomikos, 2004; Bollen et al., 2000; Cai, 1994; Engle, 1994; Haldrup and Nielsen, 2006; Hardy, 2001; Schaller and Norden, 1997; Schwert, 1989; Timmermann, 2000).

During the past two decades, several significant events occurred including the dot-com burst in 2000, the September 11 attacks in 2001, the end of the Iraq war in 2003, the Yen carry trade in 2007 and the global financial crisis in 2008, leading to abrupt jumps in stock prices and returns. Unfortunately, the regime-switching model cannot adequately describe drastic changes in prices and returns and in this paper we propose a regime-switching model with jump risks to address this limitation of the model. More specifically, we show that compared to the Black–Scholes model (BSM) (Black and Scholes, 1973) and the regime-switching model (RSM), the regime-switching model with jump risks (RSMJ) can better explain the dynamics of the S&P 500 stock index, by the estimating parameters of the Expectation–Maximization (EM) algorithm and testing these by computing the likelihood function. Subsequently, we develop a recursive formula to price a participating contract embedding a surrender option under the RSM and the RSMJ.

The remainder of the paper is organized as follows. Section 2 illustrates the framework of the participating contract, the RSM and the RSMJ for the stock index. The empirical estimates and the tests of the three models for the S&P 500 stock index are also reported in this

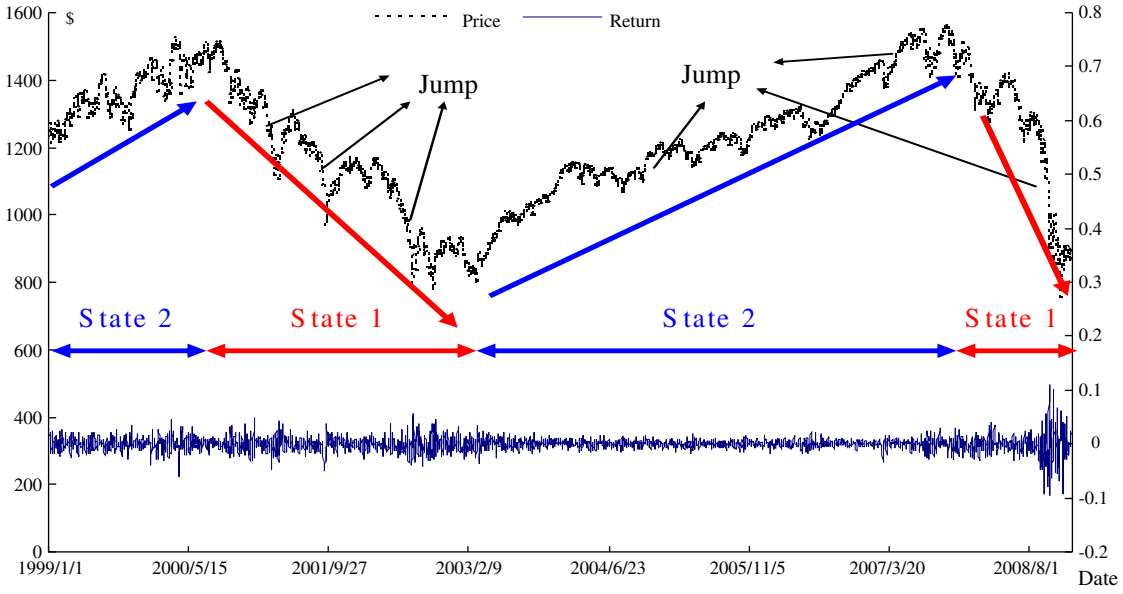

Fig. 1. The dynamics of the price and return, S&P 500 index 1999–2008.