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On decomposing inequality and poverty changes over time: A multi-dimensional decomposition

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ABSTRACT

The paper proposes a multi-dimensional decomposition of the changes in the Gini inequality index and the Sen–Shorrocks–Thon poverty index over time. The link among inequality change, re-ranking of individuals and income growth is explained by isolating the source and subgroup contributions to the determinants of the inequality change over time. We show that the poverty change over time depends on the re-ranking of individuals and the change in relative disparities among poverty gaps. These determinants of the change in poverty inequality are then decomposed by source and subgroup, yielding a three-way decomposition which combines time, source and subgroup ways of decomposing. An application to Italian household income data illustrates the proposed decomposition.

1. Introduction

In the last few years, the literature on inequality and poverty measurement has paid growing attention to combining subgroup and source ways of decomposition (Mussard and Richard, 2012; Shorrocks, 1999; Yitzhaki, 2002). Mussard and Savard (2012) showed that the decomposition by source and subgroup (the so-called multi-decomposition) can be applied to the change in the Gini inequality index over time. Mussard and Pi Alperin (2011) provided the multi-decomposition of the Sen–Shorrocks–Thon (hereafter, SST) poverty index, and also decomposed the percentage change of the index between two points in time. The changes in poverty and inequality are clearly of interest; however, they do not provide information on the movements of individuals in the income distribution and the disproportional growth between individuals' incomes over time. In this article, we extend the multi-decomposition approach to inequality and poverty changes in order to account for re-ranking between individuals and disproportional income growth in the move from an initial to a final income distribution.

Several studies investigated the link among income growth, re-ranking, and changes in inequality and poverty over time. Jenkins and Van Kerm (2006) explained the change in the (generalised) Gini index between two points in time by isolating a re-ranking component measuring the movements of individuals, and an income growth component measuring the disproportional growth between incomes. O'Neill and Van Kerm (2008) adopted the Jenkins and Van Kerm (2006) decomposition by time to explore the relationship between change in inequality and income convergence among European countries. Following the Jenkins and Van Kerm (2006) approach, Wagstaff (2009) decomposed Son's (2004) poverty growth curve. This poverty growth curve gives the growth of the mean income of the poorest $100\alpha\%$ of population. Wagstaff's decomposition isolates two components: one term measures the income growth among individuals who are initially in the poorest $100\alpha\%$ of population, the other term captures the shift of individuals from the poorest $100\alpha\%$ to the richest $100(1-\alpha)\%$ of population in the move from the initial to the final income distribution. More recently, Mussini (2013b) showed that the re-ranking and income growth components of the change in the Gini index can be decomposed by subgroup using a matrix approach based on pairwise income differences.

In this article, we refer to Mussini's matrix decomposition and extend it by adding the source decomposition dimension. Therefore, we simultaneously decompose inequality by time, source and subgroup. Using this three-way decomposition, one can detect the various source contributions to the re-ranking and total income growth effects within as well as across subgroups.

We show that the three-way decomposition can be used to decompose the change in the SST poverty index. We point out that the

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re-ranking between individuals and the change in poverty gaps determine the change in poverty inequality over time. The two poverty change determinants are then decomposed by subgroup and source, isolating the contributions of each combination between subgroup and source components to the re-ranking and poverty gap growth effects, respectively. Linking the three-way decomposition of the change in poverty inequality with the Mussard and Pi Alperin (2011) decomposition of the change in the SST index, one can observe the multi-dimensional decomposition of all the determinants of poverty change between two points in time.

The multi-dimensional decompositions of the changes in the Gini and the SST indices are applied to Italian household income data from the Survey on Household Income and Wealth conducted by the Bank of Italy (Banca d'Italia, 2012) over the 2008–2010 period. We aim at detecting the roles played by the various determinants of poverty and inequality changes between 2008 and 2010. Our findings explain that the relatively small changes in inequality and poverty reflect the interaction between re-ranking and income growth, since the disequalising effect of re-ranking and the equalising effect of income growth nearly compensate each other. Therefore, the almost unchanged poverty and inequality are not simply due to few changes in the income distribution.

The remainder of the article is organised as follows. Section 2 briefly reviews Mussini's matrix approach to the decomposition of the inequality change over time. In Section 3, the three-way decomposition of the change in inequality is shown (Section 3.1); then, the decomposition approach is extended to the SST poverty index (Section 3.2). In Section 4, the decomposition technique is applied to Italian household income data collected by the Survey on Household Income and Wealth over the 2008–2010 period. Section 5 concludes.

2. Preliminaries and notation

Consider a population of size n that is partitioned into r subgroups. Let $\mathbf{y} = (y_1,\ldots,y_n)'$ be the $n \times 1$ vector of incomes sorted in decreasing order, and $\bar{y}$ be the average income. Let 1_n be the $n \times 1$ vector with elements equal to 1. As shown in Mussini (2013a), the Gini index can be expressed as

$$G(Y) = \frac{1}{2n^2} tr\left(\mathbf{GE}'\right), \tag{1}$$

where $\mathbf{G}$ is a $n \times n$ G-matrix (a skew-symmetric matrix with upper-diagonal elements equal to -1, lower-diagonal elements equal to 1, and diagonal elements equal to 0) (Silber, 1989), and $\mathbf{E}$ is the $n \times n$ skew-symmetric matrix

$$\mathbf{E} = \frac{1}{\bar{y}}\left(\mathbf{1}_n\mathbf{y}' - \mathbf{y}\mathbf{1}_n'\right) = \begin{bmatrix} \frac{y_1 - y_1}{\bar{y}} & \cdots & \frac{y_n - y_1}{\bar{y}} \\ \vdots & \ddots & \vdots \\ \frac{y_1 - y_n}{\bar{y}} & \cdots & \frac{y_n - y_n}{\bar{y}} \end{bmatrix}. \tag{2}$$

Being y_j and y_i the j-th and i-th incomes in $\mathbf{y}$ (with $j, i =,\ldots,n$) respectively, the matrix $\mathbf{E}$ has the (i,j)-th element equal to the difference between y_j and y_i made relative to the average income $\bar{y}$. It contains the n^2 relative pairwise differences between the incomes as ordered in $\mathbf{y}$.

Now, suppose that the n individuals receive income at an initial time (hereafter, t_0) and at a final time (hereafter, t_1). Jenkins and Van Kerm (2006) decomposed the change in the Gini index between t_0 and t_1 by isolating two components: one measuring the re-ranking between individuals, the other measuring the disproportional growth between individuals' incomes in the move from t_0 to t_1.

Mussini (2013b) showed that the decomposition of the inequality change can be expressed in matrix form, and then combined the subgroup decomposition and the decomposition by time.

Let $\mathbf{y}_1 = (y_{1,1},\ldots,y_{n,1})'$ stand for the vector of t_1 incomes sorted in decreasing order, and $\mathbf{y}_0 = (y_{1,0},\ldots,y_{n,0})'$ be the vector of t_0 incomes sorted in decreasing order. Then, the matrix expression of the change in inequality over time is

$$\Delta G(Y) = G_1(Y) - G_0(Y) = \frac{1}{2n^2} tr\left(\mathbf{GE}_1'\right) - \frac{1}{2n^2} tr\left(\mathbf{GE}_0'\right). \tag{3}$$

Let $\mathbf{y}_{1|0} = (y_{1,1|0},\ldots,y_{n,1|0})'$ be the vector of t_1 incomes arranged by the decreasing order of their corresponding t_0 incomes, and $\mathbf{E}_{1|0} = (1/\bar{y}_1)\left(\mathbf{1}_n\mathbf{y}_{1|0}' - \mathbf{y}_{1|0}\mathbf{1}_n'\right)$ be the matrix containing pairwise differences between t_1 incomes as arranged in $\mathbf{y}_{10}$. The concentration index of t_1 incomes sorted by t_0 incomes is

$$C_{1|0}(Y) = \frac{1}{2n^2} tr\left(\mathbf{GE}_{1|0}'\right). \tag{4}$$

Let $\mathbf{B}$ stand for the $n \times n$ permutation matrix rearranging the elements of $\mathbf{y}_1$ to obtain $\mathbf{y}_{1|0}$, that is $\mathbf{y}_{1|0} = \mathbf{B}\mathbf{y}_1$. After some algebraic manipulations, Eq. (4) can be re-written as follows (see Mussini, 2013b, pp. 387–388),

$$C_{1|0}(Y) = \frac{1}{2n^2} tr\left(\mathbf{B}'\mathbf{GBE}_1'\right). \tag{5}$$

Adding $C_{1|0}(Y)$ as written in Eq. (4) and subtracting it as expressed in Eq. (5) to the right-hand side of Eq. (3), the matrix formulation for the decomposition of the inequality change is

$$\begin{aligned} \Delta G(Y) &= \left[\frac{1}{2n^2} tr\left(\mathbf{GE}_1'\right) - \frac{1}{2n^2} tr\left(\mathbf{B}'\mathbf{GBE}_1'\right)\right] - \left[\frac{1}{2n^2} tr\left(\mathbf{GE}_0'\right) - \frac{1}{2n^2} tr\left(\mathbf{GE}_{1|0}'\right)\right] \\ &= \frac{1}{2n^2} tr\left[\left(\mathbf{G} - \mathbf{B}'\mathbf{GB}\right)\mathbf{E}_1'\right] - \frac{1}{2n^2} tr\left[\mathbf{G}\left(\mathbf{E}_0 - \mathbf{E}_{1|0}\right)'\right] \\ &= \frac{1}{2n^2} tr\left(\mathbf{RE}_1'\right) - \frac{1}{2n^2} tr\left(\mathbf{GP}'\right) \\ &= R(Y) - P(Y). \end{aligned} \tag{6}$$

In Eq. (6), $R(Y)$ is the component which measures the re-ranking between individuals and it always produces a nonnegative contribution to the inequality change (i.e., $0 \leq R(Y) \leq 2G_1(Y)$). As we can see from the matrix expression of $R(Y)$, the movements of individuals are tracked using the matrix $\mathbf{R}$ which detects the pairs composed of individuals who re-rank in the move from t_0 to t_1. Being r_{ij} the (i,j)-th element of $\mathbf{R}$, r_{ij} equals 2 (-2) if $i > j$ $(i < j)$ and the (i,j)-th entry of $\mathbf{E}_1$, $e_{ij,1}$, is filled by the relative difference between two incomes belonging to re-ranking individuals, otherwise r_{ij} is zero. If the ranking of individuals is unchanged in the move from t_0 to t_1, then $\mathbf{B} = \mathbf{I}_n$ and $R(Y) = 0$. A simple numerical example may clarify the construction of $\mathbf{R}$. Let $\mathbf{y}_0$ be $(y_{1,0} = 8, y_{2,0} = 4, y_{3,0} = 3, y_{4,0} = 1)'$ and $\mathbf{y}_{1|0}$ be $(y_{1,1|0} = 7, y_{2,1|0} = 5, y_{3,1|0} = 6, y_{4,1|0} = 2)'$. It immediately follows that $\mathbf{y}_1$ is equal to $(y_{1,1} = 7, y_{2,1} = 6, y_{3,1} = 5, y_{4,1} = 2)'$. We can see that re-ranking occurs between the individuals whose t_1 incomes are equal to 6 and 5, respectively. This re-ranking is detected by the 4×4 matrix $\mathbf{R}$ which has $r_{32} = 2$ and $r_{23} = -2$, while its remaining elements are zero.

$P(Y)$ is the income growth component which can increase or reduce inequality between the two times. In the matrix expression of $P(Y)$ in Eq. (6), the (i,j)-th element of $\mathbf{P}$, p_{ij}, measures the change in relative disparity between the incomes of two individuals when passing from t_0 to t_1. If $p_{ij} > 0$ with $i > j$ (and $p_{ji} < 0$, since $\mathbf{P}$ is skew-symmetric), the relative disparity between t_0 incomes of two individuals is greater than the relative disparity between t_1 incomes of the same individuals; therefore, an equalising effect is ascribable to income growth between t_0 and t_1. If $p_{ij} < 0$ with $i > j$(and $p_{ji} > 0$), the relative disparity between t_0 incomes of two individuals is less than the relative disparity between