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Testing for Granger non-causality using the autoregressive metric

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ABSTRACT

A new non-causality test based on the notion of distance between ARMA models is proposed in this paper. The advantage of this test is that it can be used in possible integrated and cointegrated systems, without pre-testing for unit roots and cointegration. The Monte Carlo experiments indicate that the proposed method performs reasonably well in finite samples. The empirical relevance of the test is illustrated via an application.

1. Introduction

Since the seminal paper of Granger (1969), Granger non-causality test among economic time series have become ubiquitous in applied econometric research. This concept is defined in terms of predictability of variable x from its own past and the past of another variable y. In particularly, we say that y Granger-causes x if the past values of y can be used to predict x more accurately rather than simply using the past values of x alone. Thus, Granger causality may have more to do with precedence, or prediction, than with causation in the usual sense. However, apart from these theoretical considerations, there are a number of methodological issues arising from the various applications of Granger causality tests. It was shown that the use of non-stationary data in causality tests can yield spurious causality results (Park and Phillips, 1989; Sims et al., 1990; Stock and Watson, 1989). Thus, before testing for Granger causality, it is important to establish the properties of the time series involved. The common practice, considering for seeking of simplicity the case of 2 variables, is the following: when both series are I(0), a vector autoregressive (VAR) model in levels is used; when one of the series is found I(0) and the other one I(1), VAR is specified in the level for the I(0) variable and in terms of first difference for the I(1) variable; when both series are determined I(1) but not cointegrated, the proper model is VAR in terms of the first differences. Finally, when the series are cointegrated, we can use a vector error correction (VECM) model or a VAR model in levels. The weakness of this strategy is that incorrect conclusions drawn from preliminary analysis might be carried over onto the causality tests. An alternative method is the lag-augmented Wald test (see Dolado and Lütkepohl, 1996; Toda and Yamamoto, 1995). This method does not rely so heavily on pre-testing. However, the knowledge of the maximum order of integration is still required. Further, the lag-augmented Wald test may suffer from size distortion and low power especially for small samples (Giles, 1997; Mavrotas and Kelly, 2001).

In this paper we propose a new Granger non-causality test based on the notion of the distance between autoregressive moving average (ARMA) models, the AR metric introduced by Piccolo (1990). The advantage of this test is that it can be carried out irrespective of whether the variables involved are stationary or not and regardless of the existence of a cointegrating relationship among them. Consequently no pre-testing for unit roots and cointegration is required. Further, our test appears to be well-sized and has satisfactorily good power properties.

The remainder of the paper is organized as follows. Section 2 introduces the notion the distance between ARMA models and specifies the relationship between AR metric and Granger causality. Section 3 presents the new Granger non-causality test. Section 4 provides some Monte Carlo evidence about the finite sample behavior of our testing procedure in comparison with the lag-augmented Wald test. Section 5 contains an empirical illustration of testing causality. Section 6 gives some concluding remarks.

2. Granger causality and AR metric

The AR metric introduced by Piccolo (1990) defines the distance between two ARMA models in terms of Euclidean distance between the AR (∞) representations of the ARMA models considered. In a

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VAR framework, this notion can be connected to the concept of Granger causality considering the ARMA models implied by the VAR.

2.1. The VAR framework

Let the $n \times 1$ vector time series $\{w_t; t \in \mathbb{Z}\}$ be represented by the VAR model of order p:

$$A(L)w_t = \epsilon_t \tag{1}$$

where $A(L) = I_n - A_1L - A_2L^2 - \ldots - A_pL^p$ is an $n \times n$ matrix of polynomials in the lag operator L, and ϵ_t is a vector white noise process with positive definite covariance matrix. We assume that det $(A(z)) \neq 0$ for $|z| < 1$. This condition allows non stationarity for the series, in the sense that the characteristic polynomial of the VAR model described by equation det $(A(z)) = 0$ may have roots on the unit circle. Condition det $(A(z)) \neq 0$ for $|z| < 1$, however, excludes explicitly explosive processes from our consideration.

Consider the partition $w_t = (x_t, y'_t)'$ where x_t is a scalar time series and y_t is a $(n-1) \times 1$ vector of time series. Model (1) according to the partition of w_t, can be rewritten as:

$$\begin{bmatrix} 1-A_{11}(L) & A_{12}(L) \\ A_{21}(L) & I-A_{22}(L) \end{bmatrix} \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} \epsilon_{x_t} \\ \epsilon_{y_t} \end{bmatrix}$$
$$E\left(\begin{bmatrix} \epsilon_{x_t} \\ \epsilon_{y_t} \end{bmatrix} \begin{bmatrix} \epsilon_{x_s} & \epsilon_{y_s} \end{bmatrix} \right) = \begin{cases} \Sigma & t = s \\ 0 & t \neq s \end{cases} \tag{2}$$

where $A_{ij}(L) = \sum_{h=1}^{p} A_{ij}^{(h)} L^h$ $i, j = 1, 2$ are matrix polynomials in the lag operator L. We assume that $1 - A_{11}(L) \neq 0$ and $\det(I - A_{22}(L)) \neq 0$.

In this framework it is well known that y_t does not cause x_t (denoted by $y_t \nrightarrow x_t$) if and only if

$$A_{12}(L) = 0. \tag{3}$$

2.2. The AR metric

Let z_t be a invertible ARMA model defined as

$$\phi(L)z_t = \theta(L)\epsilon_t$$

where $\phi(L)$ and $\theta(L)$ are polynomials in the lag operator L, with no common factors, and ϵ_t is a white noise process. It is well known that this process admits the representation:

$$\pi(L)z_t = \epsilon_t$$

where the AR(∞) operator is defined by

$$\pi(L) = \phi(L)\theta(L)^{-1} = 1 - \sum_{i=1}^{\infty} \pi_i L^i.$$

Given the processes x_t and y_t following two invertible ARMA models and given their AR(∞) representations $\{\pi_{xi}\}$ and $\{\pi_{yi}\}$, the AR metric between x_t and y_t is defined as the Euclidean distance between the corresponding π-weights sequence

$$d = \left[\sum_{i=1}^{\infty} \left(\pi_{xi} - \pi_{yi} \right)^2 \right]^{\frac{1}{2}}. \tag{4}$$

The AR metric d has been widely used in time series analysis (Maharaj, 1996; Gonzalo and Lee, 1996; Grimaldi, 2004; Corduas and Piccolo, 2008; Otranto, 2008, 2010). We observe that (4) is a well defined measure because of the absolute convergence of the π-weights sequences. The asymptotic distribution of the maximum likelihood estimator $\hat{d}^2$ has been studied in Corduas (1996, 2000), D'Elia (2000) and Corduas and Piccolo (2008).

2.3. The implied ARMA models

In this subsection we introduce the ARMA models for the variable of interest x. In particular, we will consider the ARMA model implied by the VAR under the Granger non-causality condition (3) and the general ARMA model implied by the unrestricted VAR.

We note that, if the condition (3) holds, then x_t follows a univariate ARMA model given by:

$$[1 - A_{11}(L)]x_t = \epsilon_{x_t}. \tag{5}$$

The general implied ARMA model can be obtained as follows. Premultiplying both sides of (1) by the adjoint Adj $(A(L))$ of $A(L)$, we obtain

$$\det(A(L))w_t = \text{Adj}(A(L))\epsilon_t. \tag{6}$$

We note that each component of Adj $(A(L))\,\epsilon_t$ is a sum of finite order MA processes, thus it is a finite order MA process (see Lütkepohl (2005), Proposition 11.1). Hence, the subprocess $\{x_t; t \in \mathbb{Z}\}$ follows an ARMA model given by:

$$\det(A(L))x_t = \delta(L)u_t \tag{7}$$

where u_t is univariate white noise and $\delta(L)$ is an invertible polynomial in the lag operator L. More precisely, $\delta(L)$ and u_t are such that

$$\delta(L)u_t = C_1(L)\epsilon_t$$

where

$$C_1(L) = \left[\det(A(L))D(L), -\det(A(L))D(L)A_{12}(L)(I - A_{22}(L))^{-1} \right]$$

with $D(L) = [1 - A_{11}(L) - A_{12}(L)(I - A_{22}(L))^{-1}A_{21}(L)]^{-1}$ is the first row of Adj $(A(L))$. Finally, we observe that x_t has also the following autoregressive representation of infinite order:

$$\varphi(L)x_t = u_t$$

where

$$\varphi(L) = \frac{\det[A(L)]}{\delta(L)} = 1 + \varphi_1 L + \varphi_2 L^2 + \ldots$$

2.4. A characterization of the Granger non-causality

In this subsection we obtain a characterization the Granger non-causality condition (3), using the notion of distance between ARMA models measured by Eq. (4). In particular, we will consider the distance between the AR(p) model (5) and the ARMA model (7) for the subprocess $\{x_t; t \in Z\}$ implied by the VAR(p) model (1). The distance according to Eq. (4) between the models (7) and (5) is given by:

$$d = \left[\sum_{i=1}^{\infty} \left(\varphi_i - A_{11}^{(i)} \right)^2 \right]^{\frac{1}{2}}$$

where $A_{11}^{(i)} = 0$ for $i = p + 1, \ldots$

The following proposition provides a characterization of Granger non-causality.

Proposition 1. $A_{12}(L) = 0$ *if* $d = 0$.

Proof. $(\Rightarrow)$ If $A_{12}(L) = 0$, then

$$\det(A(L)) = (1 - A_{11}(L))\det(I - A_{22}(L))$$

and

$$\delta(L) = \det(I - A_{22}(L)).$$