Economic Modelling 33 (2013) 174–181

Contents lists available at SciVerse ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

An empirical estimation for mean-reverting coal prices with long memory

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ARTICLE INFO

Article history:
Accepted 5 April 2013
Available online xxxx

JEL classification:
C15
C22
C32

Keywords:
Energy model
Least squares estimation
Quadratic variations
Fractional mean-reversion processes
Monte Carlo simulation

ABSTRACT

In this paper we discuss the calibration issues of power models built on mean-reverting processes combined with long memory. The unknown parameters of fractional mean-reversion processes are estimated by a hybrid estimation method, which is built upon the marriage of the quadratic variation and the least squares. We perform a simulation study to test the efficiency of these estimators and to compare with the approach proposed by Høg (1999). Moreover, we apply our estimation procedure to some sample series of Chinese coal spot prices in real life situations. These results support the use of fractional mean-reversion processes in modeling Chinese coal prices.

1. Introduction

During the last two decades the structure of the power industry has changed dramatically worldwide. As a consequence, the search for a distribution which accurately describes the behavior of energy prices returns has generated a considerable amount of controversy. Generally when building realistic models we cannot forget about some characteristics of coal prices. There are uncertainties on coal price, coal reserve, developing cost and developing conditions, which will diminish generally with the increase of information and passage of time. At the same time the end-user demand shows high variability and strong weather and business cycle dependence. As a result, the resulting spot prices exhibit strong seasonality on the annual, weekly and daily level, as well as, mean reversion, very high volatility and abrupt, short-lived and generally unanticipated extreme price spikes or drops (Janczura and Weron, 2010; Karakatsani and Bunn, 2008). Although many scholars have presented lots of stochastic models to capture behavior of coal prices, but a problem arises: what classes of models should we then use to efficiently describe coal price dynamics?

In literate, there exist many models, which can model the behavior of coal prices. For example, Pindyck (1999) proposed a univariate model for the long-run evolution of oil, gas and coal prices by means of stochastic trends. Morana (2001) introduced a semiparametric approach for forecasting oil prices, using GARCH models and bootstrap techniques for the computation of prediction intervals. De Jong and Schneider (2009) dealt with the relationships among different series of gas and electricity prices in several European markets. Their study is very interesting since it explores the possibility of a multi-market model. Ghoshray and Johnson (2010) studied univariate models for oil, gas and coal prices, detecting structural changes in the trends which suggest that structural changes in the economy affect commodity prices in the long term. García-Martos et al. (2012) presented a comparison of several univariate and multivariate models in terms of prediction accuracy for fossil fuel and electricity prices. Among these stochastic models, it is well known that the most applied stochastic process for energy prices is the mean-reversion process.

The mean-reverting diffusion-type processes, which are also called the Vasicek model model (1977) and the CIR processes (Cox et al., 1985), were at the heart of interest rate modeling for years (Stein and Stein, 1991; Xu et al., 2011a,b). Their parsimony–often referred to as 'reduced-form'–together with their ability to represent mean reversion made them models of first choice also in coal spot price modeling (Barlow, 2002; Barz and Johnson, 1998; Geman, 2007; Kaminski, 1997; Lucia and Schwartz, 2002; Meade, 2010; Schlueter, 2010; Schwartz and Smith, 2000). An extensive review on most of the recent developments related to the parametric and other inference procedures for mean-reverting processes driven by Brownian motions can be found in Xi and Mamon (2011).

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http://dx.doi.org/10.1016/j.econmod.2013.04.015

Recently, self-similarity and long-memory have become important concepts in analyzing the time series in fields as diverse as energy, physics, chemistry, medicine, finance and environmental studies (see for example, Elder and Serletis, 2008; Elyasiani et al., 2011; Gil-Alana et al., 2010; Haldrup et al., 2010; Kang et al., 2009; Wang et al., 2011). The most popular self-similar process is the fractional Brownian motion. Indeed, the fractional Brownian motion is the only Gaussian self-similar process with stationary increments. To capture the long memory property of energy prices, it is natural to replace the standard Brownian motion in the noise process with the fractional Brownian motion (Ayadi et al., 2009; Rypdal and Løvsletten, 2013; Serletis and Andreadis, 2004). Indeed, when stochastic models driven by fractional Brownian motion are used to describe some phenomena, it is important to identify the unknown parameters of these models. As a consequence, the parameter estimation problem for stochastic models driven by fractional Brownian motion was great interest and became a challenging theoretical problem in the past decade. In the continuous-time case, statistical inference problems related to processes driven by the fractional Brownian motion have been studied extensively by many scholars (see Casas and Gao, 2008; Hu and Nualart, 2010; Kleptsyna and Le Breton, 2002; Tsai and Chan, 2005a,b; Tudor and Viens, 2007). An extensive review on most of the recent developments related to the parametric and other inference procedures for stochastic models driven by fractional Brownian motion can be found in Prakasa Rao (2010).

These papers above focused on parametric estimating of stochastic models in the continuous-time case. However, in practice it is usually only possible to observe these processes in discrete-time samples (e.g., stock prices collected once a day). Therefore, statistical inference for discretely observed processes was of great interest for practical purposes and at the same time it posed a challenging problem. Actually, in recent years, there were some attempts to investigate the inference problems of stochastic processes, when the observational scheme is discrete in nature. For instance, Tudor and Viens (2007) treated estimation problem of the drift parameter in stochastic models associated with fractional Brownian motion using the following idea: first, one constructs an estimator in the continuous time model and then the continuous estimator is discretized. Some author (see, for example, Bertin et al., 2011; Hu et al., 2011; Xiao et al., 2011a,b) considered the problem of estimating coefficients in some models driven by fractional Brownian motion based on an approximation by a discretized stochastic differential equation.

Although there are papers focused on modeling coal prices, the literature is scarce on attempts to consider the "mean reversion" property together with the long memory property. The aim of this paper is to show how to test the long memory in energy time series and to determine the unknown parameters in fractional mean-reversion processes. As a consequence, the main contribution of this paper is twofold. First, we present a model that captures the most important characteristics of coal spot prices such as mean reversion and long memory. Moreover, we estimate the unknown parameters in fractional mean-reversion processes, which are observed with random noise errors in discrete framework. Second, we describe the numerical implementation based on our method. The comparison of our estimation method and Høg's method (1999) illustrates that the method proposed in this paper is efficient. In addition, we show how to apply our approach in realistic contexts.

The paper is organized as follows. In Section 2, we present the model and provide the estimation procedure. Some numerical experiments, which illustrate the performance of the proposed method, are presented in the latter part of this section. Moreover, the comparisons of our method with Høg's method are also provided in this section. Section 3 presents our empirical results of Qinhuangdao gifted mixed Datong coal, Qinhuangdao gifted mixed Shanxi coal, Qinhuangdao large mixed Shanxi coal and Qinhuangdao gifted mixed general coal. Finally, Section 4 draws the concluding remarks.

2. Model, methodology and simulation results

In the last few years there has been a rapidly increasing literature on stochastic models for prices of coal and other commodities. Many researchers have observed that the models typically used in financial markets are inappropriate due to the special features of commodity prices and especially of coal prices. Coal can be bought in the spot market, but once purchased it must be used, since in most cases coal is stored not cheaply. Moreover, coal prices have the tendency to revert rapidly from price spikes to a mean level. Characteristic times of mean reversion have a magnitude of days or at most weeks and can be explained with changes of weather conditions or recovery from power plant outages. In addition, due to the increasing uncertainty about factors such as supply and demand or fuel costs in the long-term future, a long memory model seems more appropriate. Hence we have concluded that two distinctive characteristics of coal markets should be accounted for in the model: the mean reversion of the price and the long-range dependence.

2.1. The model

Schwartz (1997) accounts for the mean reversion, and Lucia and Schwartz (2002) extend the mean reverting model to account for a deterministic seasonality. In fact, the mean-reversion process is one of the most applied stochastic processes for coal prices. As logarithms of the coal prices are modeled to reach variance stabilization, the mean-reversion process or the so-called Ornstein–Uhlenbeck process can be formulated for the price changes with the following stochastic differential equation:

$$dS_t = (\lambda - \theta S_t)dt + \sigma dW_t \ , t \geq 0, S_0 = 0, \tag{2.1}$$

where S_t denotes the spot market coal price at time t, the first term of the mean reversion process describes the so-called drift component $(\lambda - \theta S_t)dt$. The parameter λ/θ determines the "reversion speed" of the stochastic component to their long-term mean λ. The economic interpretation of this mean-reversion component is that stochastic price fluctuations around the mean and price peaks are only temporarily, caused by e.g. power plant outages or capacity shortages. The second term, the stochastic component σdW_t in fact, corresponds to the standard Brownian motion.

Let us mention that the model (2.1) does not incorporate long memory. Actually, a recent empirical work of Tabak and Cajueiro (2007) has founded a fractal structure for crude oil prices including Brent and WTI. Recently, empirical studies on energy industry revealed the most important characteristic, long-term memory (see, for example, Barros et al., 2012; Lean and Smyth, 2009). Fortunately, in order to make the models more realistic, much attention has been given to long-range dependence. At the same time alternative models entered into consideration. In particular, models using the fractional Brownian motion were developed. Consequently, the fractional Brownian motion and processes based on it have found many applications in fields as diverse as energy, physics and environmental studies. And hence we propose in this paper a similar model extended to account for the long memory property. In other words, based on Eq. (2.1), we shall consider the following special fractional processes, namely fractional mean-reverting processes, which satisfy the following stochastic differential equation

$$dS_t = (\lambda - \theta S_t)dt + \sigma dB_t^H \ , \ t \geq 0, S_0 = 0, \tag{2.2}$$

where S_t denotes the spot market price at time t, the parameters θ, λ and σ are constants, $(B_t^H, t \geq 0)$ is a fractional Brownian motion with Hurst parameter $H > 1/2$ on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with a filtration $\{\mathcal{F}\}_{t \geq 0}$.

Once the stochastic model for spot coal price is fully specified, we are left with the problem of calibrating it to market data. In this paper,