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Dynamic asset pricing model with heterogeneous sentiments

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ABSTRACT

The systematic and important role of investor sentiment has been supported by some recent empirical and theoretical literatures. In this paper, we present a dynamic asset pricing model with heterogeneous sentiments and we find that the equilibrium stock price is the wealth-share-weighted average of the stock prices that would prevail in an economy with one sentiment investor only. Moreover, heterogeneous sentiments induce fluctuations in the wealth distribution, which increases stock return volatility and induces mean reversion in stock returns. The model offers a partial explanation for the financial anomaly of mean reversion.

1. Introduction

Traditional asset pricing theory suggests that economists can safely ignore individual irrational behavior at the aggregate level (Friedman, 1953). But this argument is not adopted by behavioral finance, which argues that the investment strategy may be impacted by investor noise, investor psychology, or investor sentiment. Some noise trader models are proposed to illustrate the influence of noise on the stock price (De long et al., 1990; Yan, 2010). For example, Yan (2010) presented a noise model, where individual biases often cannot be cancelled out by aggregation. The shortcoming of noise models is that the noise information is difficult to be identified and cannot be measured, consequently can't be empirically testified. Yang and Yan (2011) suggested that investor sentiment is easy to be measured by variant methods and the related result is supported by some financial experiments (Statman et al., 2008).

Nowadays, the systematic role of investor sentiment has been supported by some empirical and theoretical studies. A number of empirical studies have shown that investor sentiment has a systematic impact on stock return (Baker and Wurgler, 2006, 2007; Baker et al., 2012; Brown and Cliff, 2004; Kumar and Lee, 2006; Kurov, 2010; Lee et al., 1991; Liao et al., 2011; Schmeling, 2009; Verma and Soydemir, 2009; Yang and Zhang, 2013a, 2013b; Yu and Yuan, 2011).

Some static asset pricing models have been developed to support the role of investor sentiment, such as Yang et al. (2012), Yang and Yan (2011), and Yang and Zhang (2013a). Yang and Yan (2011) showed that the excess return is negatively related to a high sentiment bigger than a critical point, but positively related to a high sentiment smaller than this point. Yang et al. (2012) presented a sentiment capital asset pricing model, and showed that investor sentiment is a nonlinear systematic factor for asset pricing. Yang and Zhang (2013a) presented a sentiment asset pricing model with consumption, and showed that the stock price has a wealth-weighted average structure and the investor's wealth proportion could amplify the sentiment shock on the asset price.

These static asset pricing models have showed the systematic impact of the investor sentiment on the stock price. However, compared with only one transaction, investors frequently trade stocks in capital market. Moreover, static models usually assumed that investor sentiment is a constant. In fact, investors usually update their sentiment upon receiving more data. For example, the magnitude of investor sentiment decreases over time when investors are learning over time. Hence, the dynamic characteristic of stock price model is closer to the reality capital market and the setting of dynamic sentiment asset pricing model could explain the complex of the stock price changing form.

Yang and Zhang (2013b) presented a dynamic sentiment asset pricing model with one representative investor, and showed that investor sentiment has a significant impact on the equilibrium stock price. Moreover, time varying sentiments can lead to a complex road of the price change. There is only one representative investor in the model, but there are many types of investors in the reality stock market, such as rational investors, optimistic investors and pessimistic investors etc. To our knowledge, the dynamic asset pricing models with heterogeneous sentiments have not been developed.

Based on the framework of consumption-based model, we present a dynamic asset pricing model with heterogeneous sentiments. The

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dynamic sentiment asset pricing model shows that heterogeneous sentiments have a significant impact on the equilibrium stock price.

First, to examine Friedman's (1953) argument that rational investors can cancel irrational investors out, let us consider an economy with one rational investor and one sentiment investor. The model shows that the wealth share of the sentiment investor is more than the one of the rational investor when investor sentiment is close to the stock's performance. This result is not consistent with the conventional argument.

Second, in an economy with one optimistic investor and one pessimistic investor, the impact of optimistic sentiment on stock price could be partly offset by pessimistic sentiment. The net impact of heterogeneous sentiments depends on the fraction of investor wealth and the sentiment level, and is not equal to zero in general. Hence, investor sentiment still has an effect on the equilibrium.

Finally, in an economy with many investors, the equilibrium stock price is the wealth-share-weighted average of the stock prices that would prevail in an economy with one sentiment investor only. Moreover, heterogeneous sentiments induce fluctuations in the wealth distribution, which increases the stock return volatility and induces mean reversion in stock returns. The model offers a partial explanation for the financial anomalies of mean reversion.

The rest of this paper is organized as follows. Section 2 presents a dynamic model with heterogeneous sentiments. Section 3 presents the equilibrium stock price. Section 4 is the discussion of the model. Section 5 concludes.

2. The dynamic model

2.1. The economy

Our goal is to set up a dynamic model with heterogeneous sentiments to formalize the discussions outlined in the introduction. In this paper, we adopt the framework of consumption-based asset pricing model. One advantage of this dynamic model is that it is a general analytical framework including CAPM, APT and MPT.

Consider a two-period (three dates) model with $t = 0,1,2$. There are two tradable assets in the economy: a riskless bond and a stock. Given that the bond is assumed to be in perfectly elastic supply, and the interest rate r_f is set exogenously. The stock, normalized to one share, is a claim to two positive dividends D_1 and D_2. For $t = 1,2$,

$$\ln D_t = \ln D_{t-1} + g_t^D$$

where g_1^D and g_2^D are independent and identically distributed (i.i.d.) with a distribution $N(\mu_D,\sigma_D^2)$ and $D_0 = 1$.

The exogenous aggregate consumption supply at time t is given by C_t, with $C_0 > 0$, and for $t = 1,2$,

$$\ln C_t = \ln C_{t-1} + g_t^c$$

where g_1^c and g_2^c are identically distributed with a distribution $N(\mu_c,\sigma_c^2)$. For simplicity, the shocks to dividends g_t^D and the shocks to consumption g_t^c are assumed to be independent.

2.2. Investor sentiment

That is an economy with N consumers, no share of stock and no bond. The systematic role of investor sentiment on stock returns has been proposed by some empirical and theoretical literatures (Baker and Wurgler, 2006; Brown and Cliff, 2004; Kumar and Lee, 2006). For example, Baker and Wurgler (2006) employed principal component analysis to construct a composite market sentiment index. They selected six variables, such as closed-end fund discount, turnover rate, number of IPO, first-day return of IPO, equity share in new issues, and dividends, as the proxies of market aggregate sentiment. They argued that this method can include more information of stock return. We now study the case in which consumers are affected by the investor sentiment.

In the market, investors would perceive stock dividend with heterogeneous sentiments. For example, sentiment investors overestimate the stock dividend, and other investors underestimate the stock dividend. Suppose that investor i is pessimistic (or optimistic) about the mean growth rate:

$$\mu_D^i = \mu_D + f(SI_i) \tag{1}$$

where μ_D^i is a function of growth rate, SI_i is investor i's sentiment and $f(\cdot)$ is a sentiment function. Eq. (1) means that $g_1^D(SI_i)$ and $g_2^D(SI_i)$ are independent and identically distributed (i.i.d.) with a distribution $N(\mu_D^i,\sigma_D^2)$ in sentiment investor's opinion.

Investor sentiment SI_i is an independent realization from S, which is a random variable and is distributed with mean $\overline{S}$ and variance σ_S^2. The random variable S is independent of all fundamentals. If investor i is rational then $SI_i = 0$. In general, the mean $\overline{S}$ is not equal to zero according to the results of empirical and financial experiment researches (Baker and Wurgler, 2006; Kumar and Lee, 2006; Statman et al., 2008). The mean sentiment $\overline{S}$ is a measure of the average "bullishness" of the sentiment trader. For example, $\overline{S} > 0$ corresponds to the case in which investor sentiment is optimistic by aggregation in the bull market, and $\overline{S}<0$ corresponds to the case in which investor sentiment is pessimistic by aggregation in the bear market.

The sentiment function $f(\cdot)$ is excess growth rate about the mean growth rate, and satisfies these properties as follows: (1) $f(\cdot) > 0$ when investor sentiment is optimistic ($SI_i > 0$); (2) $f(\cdot) < 0$when investor sentiment is pessimistic ($SI_i < 0$); (3) $f(\cdot) = 0$when investor sentiment is rational ($SI_i = 0$).

2.3. The optimization consumption problem

Assuming investor i has the logarithm preference, which means that the utility function $U(\cdot)$ is $\ln(\cdot)$. Investor i's optimization consumption problem is

$$\max E_0^i\left[\sum_{t=0}^{2}\beta^t \ln\left(C_t^i\right)\right], 0<\beta<1,$$

where β is the patience parameter, and C_t^i is the consumption of sentiment investor i at time t. Budget constraint can be written as

$$W_{t+1}^i = r_{t+1}\left(W_t^i - C_t^i\right), \quad t = 0,1$$

where W_t^i is the investor i's wealth at time t, and r_{t+1}^s is the sentiment stock return at time $t+1$. Denote the equilibrium stock price at time t as P_t. The sentiment return r_t^s is given by

$$r_1^s = \ln((D_1+P_1)/P_0) = \ln\left(\left(D_0 \times e^{g_1^D(SI_i)} + P_1\right)/P_0\right)$$

$$r_2^s = \ln(D_2/P_1) = \ln\left(D_1 e^{g_2^D(SI_i)}/P_1\right)$$

where $E(g_1^D(SI_i)) = E(g_2^D(SI_i)) = \mu_D + f(SI_i)$.

The definition of a competitive equilibrium is standard: In the equilibrium, sentiment investor maximizes the objective function and the good and financial markets clear. The standard consumption-based asset pricing model implies that the stock prices are given by the rational Euler equation:

$$1 = E\left(\beta(C_{t+1}/C_t)^{-1} r_t\right).$$