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A general to specific approach for constructing composite business cycle indicators☆

Gianluca Cubadda [a,*], Barbara Guardabascio [b], Alain Hecq [c]

[a] Università di Roma "Tor Vergata", Department of Economics and Finance, Via Columbia 2, 00133 Roma, Italy
[b] ISTAT, DCSC-SER/C, Viale Liegi 13, 00198 Roma, Italy
[c] Maastricht University, Department of Quantitative Economics, P.O. Box 616, 6200 MD Maastricht, The Netherlands

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ABSTRACT

Combining economic time series with the aim to obtain an indicator for business cycle analyses is an important issue for policy makers. In this area, econometric techniques usually rely on systems with either a small number of series, *N*, or, at the other extreme, a very large *N*. In this paper we propose tools to select the relevant business cycle indicators in a "medium" *N* framework, a situation that is likely to be the most frequent in empirical works. An example is provided by our empirical application, in which we study jointly the short-run co-movements of 24 European countries. We show, under not too restrictive conditions, that parsimonious single-equation models can be used to split a set of *N* countries in three groups. The first group comprises countries that share a synchronous common cycle, a non-synchronous common cycle is present among the countries of the second group, and the third group collects countries that exhibit idiosyncratic cycles. Moreover, we offer a method for constructing a composite coincident indicator that explicitly takes into account the existence of these various forms of short-run co-movements among variables.

1. Introduction

Building a composite business cycle indicator from a set of *N* economic time series is, per se, not too difficult. One can simply either average the relevant individual indicators or combine them using factor models, see inter alia Stock and Watson (1989) and Forni et al. (2001). In practice, it is not obvious that more elaborated methods produce more accurate results. For instance, it is illustrated in Hecq (2005) that randomly generated linear combinations of the four coincident indicators used by The Conference Board provide with composite indicators that deliver very similar turning points of the US economic activity. However, an improvement in forecasting the business cycles is observed for methods that explicitly take into account the existence of short-run co-movements among the individual business cycle indicators (Cubadda, 2007a).

This paper contributes to the literature on the identification of common cycles and the construction of composite business cycle indicators in two ways.

First, we provide methods for selecting the individual cyclical indicators that are based on the detection of common cycles among variables. Indeed, prior to the building of any composite business cycle indicator, we propose to dig deeper into the detection of groups of variables that are homogenous with regard to the presence of short-run co-movements. For instance, let us consider the empirical investigation that we have in this paper, namely the analysis of GDP growth rates of 24 European countries. Several studies have emphasized the existence of business cycle co-movements among European economies, see the survey by De Haan et al. (2008) and the references therein. Our main concern in this paper is to develop a strategy aiming at splitting those *N* countries into three groups of respectively N_1, N_2, and N_3 time series. These three clusters will be obtained thanks to a measure of the degree of cyclical commonality among the various economies. In particular, the first group is such that there is a common synchronous cycle among these N_1 time series (Engle and Kozicki, 1993; Vahid and Engle, 1993); the N_2 variables of the second group share a non-synchronous common cycle (Cubadda and Hecq, 2001), and the last group comprises N_3 series with idiosyncratic short-run dynamics. For small dimensional systems, a VAR analysis with additional reduced rank restrictions can be undertaken to discover these groups (Cubadda, 2007b). However, this strategy is unfeasible when *N* is too large relatively to the number of observations *T*. Hence, we provide some mild assumptions under which each equation of a VAR is endowed with a factor structure. The main attractive features of this approach are twofold: (*i*) the presence of the various kinds of co-movements is determined using

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* Corresponding author.
E-mail addresses: gianluca.cubadda@uniroma2.it (G. Cubadda), guardabascio@istat.it (B. Guardabascio), a.hecq@maastrichtuniversity.nl (A. Hecq).
URL's: http://www.economia.uniroma2.it/nuovo/facolta/docenti/curriculum/GianlucaCubadda.html (G. Cubadda), http://www.personeel.unimaas.nl/a.hecq/ (A. Hecq).

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only parsimonious single-equation models; (*ii*) these models can be specified according to the general-to-specific methodology; see, inter alia, Campos et al. (2005). In particular, one can rely on automatic selection procedures such as those already implemented in Gretl or in OxMetrics for instance.

Second, after having determined these groups, we offer a method of constructing the "best" composite coincident indicator that explicitly takes into account the existence of these various forms of short-run co-movements among variables. In particular, series from the second group are preliminarily "aligned" in order to display a common synchronous cycle with the variables of the first group. Then we exploit the common cycle property in order to build a unique composite coincident indicator.

The paper is organized as follows. Section 2 presents our new method for investigating the presence of different kind of co-movements in a set of N time series, N being too large to rely on usual multivariate time series tools. Section 3 evaluates our procedure in the light of a Monte Carlo analysis. We compare automatic selection procedures based either on sequential Wald tests or on information criteria. Section 4 is dedicated to analyze co-movements among 24 European countries, build the composite coincident indicator, and compare it with already existing indicators. Section 5 concludes.

2. Identification of business cycle co-movements

2.1. Synchronous and non-synchronous common cycles

Let $Y_t \equiv (y_{1t},\ldots,y_{Nt})'$ denote the N-vector of the time series of interest. We assume that Y_t is generated by the following stationary vector autoregressive model of order p (VAR(p) hereafter):

$$\Phi(L)Y_t = \varepsilon_t, t = 1,2,\ldots,T, \tag{1}$$

where $\Phi(L) = I_n - \sum_{j=1}^{p} \Phi_j L^j$ and ε_t are i.i.d. innovations with $E(\varepsilon_t) = 0$, $E\left(\varepsilon_t \varepsilon_t'\right) = \Sigma$ (positive definite) and finite fourth moments.

In this framework, serial correlation common feature (SCCF hereafter, see Engle and Kozicki, 1993) holds if there exists a full-rank $(N \times s)$-matrix δ $(s < N)$, whose columns span the cofeature space, such that

$$\delta' Y_t = \delta' \varepsilon_t \tag{2}$$

is a s-dimensional zero mean vector innovation process with respect to the information available at time $t-1$. Consequently, SCCF arises if there exists a matrix δ such that the conditions $\delta'_j \Phi_j = 0_{(s\times N)}$, $j = 1,\ldots, p$ are jointly satisfied.

Notice that under SCCF, the VAR model (1) can be rewritten as the following reduced-rank regression model

$$Y_t = \delta_\perp \sum_{j=i}^{p} A'_j Y_{t-j} + \varepsilon_t \equiv \delta_\perp X_{t-1} + \varepsilon_t,$$

where $\delta'\delta_\perp = 0$ and A_j is a $(N - s) \times s$ matrix for $j = 1,\ldots, p$. Since all the predictable fluctuations of series Y_t are due to the $(N - s)$ common dynamic factors X_{t-1}, the existence of SCCF is equivalent to the presence of synchronous common cycles among series Y_t.

Moreover, Vahid and Engle (1993) show that if series Y_t are the first differences of I(1) variables, condition (2) is equivalent to the presence of $(N - s)$ common components among the deviations of the series levels from their random walk trends. Hence, the notion of SCCF could in principle be used to construct composite business cycle indicators based on both the traditional notions of "growth cycle" (i.e. fluctuations in the economic activity around a long-run trend) and "growth rate cycle" (i.e. fluctuations of the growth rate of economic activity). Within this paper we follow the growth rate cycle approach mainly because the limited time span of our data prevents a statistically credible analysis of their long-run properties.

In order to allow for adjustment delays, Cubadda and Hecq (2001) propose to look at the presence of non-synchronous common cycles in the context of the polynomial serial correlation common feature (PSCCF hereafter) modeling. In this framework there exists a full-rank $(N \times s)$ matrix δ_0 such that under the null hypothesis that PSCCF of order m $(1 \le m < p)$ holds if the conditions $\delta'_0\Phi_h \neq 0_{(s\times N)}$, $h = 1,\ldots, m$, and $\delta'_0\Phi_j = 0_{(s\times N)}$, $j = m + 1,\ldots, p$ are jointly satisfied. This is equivalent to requiring that there exists a polynomial matrix $\delta(L) = \delta_0 - \sum_{h=1}^{m} \delta_h L^h$ such that

$$\delta'(L)Y_t = \delta'_0\varepsilon_t,$$

where $\delta_h = \Phi'_h\delta_0$, $i = 1,\ldots, m$.

Under PSCCF, the VAR model can be rewritten as the following partially reduced-rank regression model

$$Y_t - \sum_{h=1}^{m} \Phi_h Y_{t-h} = \delta_{0\perp} \sum_{j=m+1}^{p} A'_{0j} Y_{t-j} + \varepsilon_t \equiv \delta_{0\perp} X_{0t-1} + \varepsilon_t,$$

where A'_{0j} is a $(N - s) \times s$ matrix for $j = m + 1,\ldots, p$, which reveals that series Y_t share common dynamics after m periods.

Issler and Vahid (2006) and Cubadda (2007a) discuss how to obtain composite cyclical indicators under, respectively, SCCF and PSCCF. For instance, Issler and Vahid (2006) look at the linear combinations $\delta'_\perp Y_t$. In Section 5 we extend this approach to the case that both SCCF and PSCCF are present in the data.

2.2. A joint determination of the groups

Let us now assume that there exists a partition $Y_t = [Y_{1t}, Y_{2t}, Y_{3t}]$, where the N_1 series Y_{1t} share a synchronous common cycle, the N_2 series Y_{2t} share a non synchronous common cycle, and the remaining series Y_{3t} present idiosyncratic short-run dynamics. According to the definitions provided in the previous section, this is equivalent to assuming that series Y_{1t} are characterized by the presence of $(N_1 - 1)$ SCCF vectors, series Y_{2t} exhibit $(N_2 - 1)$ PSCCF vectors and the remaining series Y_{3t} do not present any SCCF or PSCCF. With the researcher being not aware of this partition, the goal is to find out the series that are co-moving and belong to sets Y_{1t} and Y_{2t}. This new strategy is developed in this section.

Since we have assumed that series Y_t are generated by a stationary VAR(p) model, each series Y_{it} follows the stable dynamic regression model

$$y_{it} = \phi_{ii}(L)y_{it-1} + \sum_{k\neq i}^{N} \phi_{ik}(L)y_{kt-1} + \varepsilon_{it}, \quad i = 1,\ldots,N; t = 1,\ldots,T, \tag{3}$$

where $\phi_{it}(L)$ and $\phi_{ik}(L)$ are scalar polynomials of order $(p - 1)$, and ε_{it} is i-th element of the innovation vector ε_t.

A statistical issue arises when the number of regressors, $N \times p$, becomes too large with respect to the sample size T. For instance, in our empirical application we have $N = 24$ and $T = 56$, which implies that it is not even feasible to estimate the unrestricted model (3) by OLS for $p > 2$. Hence, we further assume that

$$\sum_{k\neq i}^{n} \phi_{ik}(L)y_{kt} = \beta_i(L) \sum_{k\neq i}^{n} \omega_k y_{kt}, \tag{4}$$

where $\beta_i(L)$ is a scalar polynomial of order $p - 1$ and ω_i is a scalar for $i = 1,\ldots, N$.

Notice that this is equivalent to postulating the following factor-augmented autoregressive (FAAR) structure for each series y_{it}

$$y_{it} = \alpha_i(L)y_{it-1} + \beta_i(L)f_{t-1} + \varepsilon_{it}, \tag{5}$$

where $\alpha_i(L) = [\phi_{ii}(L) - \beta_i(L)\omega_i]$ and the common factor is $f_t = \sum_{k=1}^{n} \omega_k y_{kt}$. In Section 4 we discuss various alternatives for