Economic Modelling 37 (2014) 53–64

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

Optimal risk and dividend control problem with fixed costs and salvage value: Variance premium principle

Dingjun Yao [a], Hailiang Yang [b,*], Rongming Wang [c]

[a] School of Finance, Nanjing University of Finance and Economics, Nanjing 210023, PR China
[b] Department of Statistics and Actuarial Science, The University of Hong Kong, Hong Kong, PR China
[c] School of Finance and Statistics, East China Normal University, Shanghai 200241, PR China

ARTICLE INFO

Article history:
Accepted 22 October 2013

Keywords:
Dividend payment
Capital injection
Proportional reinsurance
Variance principle
Fixed costs
Salvage value

ABSTRACT

In this paper we study the combined optimal dividend, capital injection and reinsurance problems in a dynamic setting. The reinsurance premium is assumed to be calculated via the variance principle instead of the expected value principle. The proportional and fixed transaction costs and the salvage value at bankruptcy are included in the model. In both cases of unrestricted dividend rate and restricted dividend rate, we obtain the closed-form solutions of the value function and the optimal joint strategies, which depend on the transaction costs and the profitability in future.

1. Introduction

The classical optimal dividend problem for an insurance company consists in finding a dividend payment strategy that maximizes the total expected discounted dividends until the bankruptcy time. Much research on this issue has been carried out for various surplus process models. See, Asmussen and Taksar (1997), Gerber and Shiu (2006), Avram et al. (2007), Belhaj (2010), and Azcue and Muler (2012). Capital injection is one possible way to help the manager to run the business. The company sometimes needs to raise new capitals from the market in order to continue the business. Some papers assume that the company can survive forever with forced capital injections. The expected cumulative discounted dividends minus the expected discounted costs of capital injections can be regarded as the company's value, the management seeks to find the join optimal dividend payment and capital injection strategies that maximize this value. There are many papers on this topic, for instance, Sethi and Taksar (2002), Avram et al. (2007), Kulenko and Schmidli (2008) and Yao et al. (2011). However, capital injection is not always profitable when the company is facing the financial difficulty. L∅kka and Zervos (2008) study the combined optimal dividend and capital injection problem by taking into account the possibility of bankruptcy. The optimal strategy happens to be either a dividend barrier strategy without capital injections, or another dividend barrier strategy with forced injections when surplus is null to prevent bankruptcy, which depends on the parameters of risk model. By adopting their technique, some extended results are obtained in other risk models. See, He and Liang (2009), Dai et al. (2010) and Yao et al. (2010).

Reinsurance is an effective tool for insurance companies to control the risk exposure. Due to its practical importance and theoretical value, some researchers begin to pay attention to the combined dividend and reinsurance problem. Some literature on this issue includes Asmussen and Taksar (1997), Choulli et al. (2003), H∅gaard and Taksar (2004), Cadenillas et al. (2006), Meng and Siu (2011) and Peng et al. (2012). As we can see, in this literature, the expected value principle is commonly used as the reinsurance premium principle due to its simplicity and popularity in practice. Although the variance principle is another important premium principle, very few papers consider using it for risk control in a dynamic setting. Zhou and Yuen (2012) first study the optimal dividend and capital injection problem with reinsurance under the variance premium principle. Depending on whether there exist restrictions on dividend rates, they provide the optimal joint strategies in two different cases, the proportional costs for capital injections are also considered. In this paper, we continue studying the optimal dividend, capital injection and reinsurance problem with variance premium principle in a dynamic setting. Comparing with the work of Zhou and Yuen (2012), we add the fixed costs for capital injections and a salvage value at the time of bankruptcy in our model. In real financial market, transaction cost is an unavoidable issue, especially, the fixed cost (for example, advisory and consulting fees) can generate some difficult impulse control problems. See, for example, Paulsen (2008), Bai et al. (2010), Meng and Siu (2011) and Yao et al. (2011). The salvage value of the insurer can be explained as an insurer's brand name or agency network. As we know, very little work considers optimal dividend strategies under a salvage (or penalty) for bankruptcy. A

* Corresponding author. Tel.: +852 28578322; fax: +852 28589041.
E-mail addresses: yaodingjun@sina.cn (D. Yao), hlyang@hku.hk (H. Yang), rmwang@stat.ecnu.edu.cn (R. Wang).

0264-9993/$ – see front matter
http://dx.doi.org/10.1016/j.econmod.2013.10.026

few examples are Taksar (2000), Gerber et al. (2006), Thonhauser and Albrecher (2007), Loeffen and Renaud (2010) and Liang and Young (2012). By including the fixed costs and salvage value, our model is more realistic. Under some new objective functions, we present the associated optimal joint dividend, capital injection and reinsurance strategies. We show, under our model, that the decision to declare bankruptcy or to collect new capitals depends on the model parameters, which is consistent with the results and idea in LØkka and Zervos (2008).

The outline of this paper is as follows. In Section 2, we introduce the framework of this paper and formulate two general optimization problems concerning with dividend payments, capital injections and reinsurance under the variance premium principle. In Section 3, we consider two suboptimal problems in the cases of unrestricted and restricted rates of dividend payments with forced capital injections. In Section 4, a similar study is carried out for two suboptimal problems without considering capital injections. Finally, by comparing the solutions of suboptimal problems, we identify the closed-form solutions to the general optimal problems in Section 5, which depend on the relationships among the parameters of risk model.

2. Model formulation and the optimal control problem

We first introduce the framework of this paper. Let $(\Omega, \mathcal{F}, \mathrm{P})$ be a probability space with the filtration $\{\mathcal{F}_t\}$ satisfying the usual conditions. We first present the classical insurance risk model of an insurance company, which means the surplus of an insurance company can be modeled by

$$U_t = x + ct - \sum_{n=1}^{N_t} Y_n,$$

where x is the initial surplus, c is the premium rate, N_t is a Poisson process with constant intensity λ, and random variables Y_n's are positive *i.i.d.* claims with finite mean μ_1 and finite second moment μ_2^2. A reinsurance contract can be represented by a measurable functional $\mathrm{R}(\cdot)$ defined on the space composed of all positive random variables such that $0 \le R(Y) \le Y$. Under reinsurance R, a positive risk Y is decomposed into two parts, namely $R(Y)$ and $Y - R(Y)$, where $R(Y)$ is retained by the insurer and $Y - R(Y)$ is ceded to the reinsurer. Suppose that reinsurance R is taken for each claim. Then the total ceded risk up to time t is given by $\sum_{n=1}^{N_t}(Y_n - R(Y_n))$, and the aggregate reinsurance premium under the variance principle takes the form

$$\mathrm{E}\left(\sum_{n=1}^{N_t}(Y_n - R(Y_n))\right) + \theta \mathrm{D}\left(\sum_{n=1}^{N_t}(Y_n - R(Y_n))\right) = \lambda\left[\left(\mu_1 - \mathrm{E}(R(Y_1)) + \theta \mathrm{E}\left((Y_1 - R(Y_1))^2\right]t,\right.$$

where E and D stand for expectation and variance, respectively, and $\theta > 0$ is a loading associated with the variance of ceded risk. Then the premium process in the presence of reinsurance R can be written as

$$U_t^R = x + \left(c - c^R\right)t - \sum_{n=1}^{N_t} R(Y_n), \tag{2.1}$$

where $c^R = \lambda[(\mu_1 - \mathrm{E}(R(Y)) + \theta \mathrm{E}((Y - R(Y))^2]$ represents the reinsurance premium rate associated with R. Here we assume that the reinsurance market is frictionless. This means that the reinsurance premium rate is equal to the premium rate $c = \lambda(\mu_1 + \theta\mu_2^2)$ if the whole risk is ceded to the reinsurer. We approximate the model (2.1) by a pure diffusion model $\{X_t^R, t \ge 0\}$ with the same drift and volatility. Specifically, X_t^R satisfies the following stochastic differential equation

$$X_t^R = x + \int_0^t \theta\lambda\left(\mu_2^2 - \mathrm{E}(Y_1 - R(Y_1))^2\right)\mathrm{d}s + \int_0^t \sqrt{\lambda \mathrm{E}\left((R(Y_1))^2\right)}\mathrm{d}B_s, \tag{2.2}$$

with $X_0^R = x$, where $\{B_t, t \ge 0\}$ is a standard Brownian motion, adapted to the filtration $\mathcal{F}_t^B := \sigma\{B_s; 0 \le s \le t\}$. From now on, R is assumed to be a proportional reinsurance policy with $R(y) = (1 - a)y$. Then we represent Eq. (2.2) as

$$X_t^a = x + \int_0^t \left(1 - a^2\right)\theta\lambda\mu_2^2 \mathrm{d}s + \int_0^t (1 - a)\sqrt{\lambda}\mu_2 \mathrm{d}B_s, \tag{2.3}$$

with $X_0^a = x$.

Suppose that the proportion a can be adjusted dynamically to control the risk exposure. Denote L_t as the cumulative amount of dividends paid from time 0 to time t. The capital injection process $\left\{G_t = \sum_{n=1}^{\infty} I_{\{\tau_n \le t\}}\eta_n\right\}$ is described by a sequence of increasing stopping times $\{\tau_n, n = 1,2\ldots\}$ and a sequence of random variables $\{\eta_n, n = 1,2,\ldots\}$, which represent the times and the sizes of capital injections, respectively. A control strategy π is described by $\pi = (a^\pi; L^\pi; G^\pi) = (a^\pi; L^\pi; \tau_1^\pi, \cdots \tau_n^\pi, \cdots; \eta_1^\pi, \cdots, \eta_n^\pi, \cdots)$. The controlled surplus process associated with π is given by

$$X_t^\pi = x + \int_0^t \left(1 - \left(a_s^\pi\right)^2\right)\theta\lambda\mu_2^2 \mathrm{d}s + \int_0^t \left(1 - a_s^\pi\right)\sqrt{\lambda}\mu_2 \mathrm{d}B_s - L_t^\pi + \sum_{n=1}^{\infty} I_{\{\tau_n^\pi \le t\}}\eta_n^\pi. \tag{2.4}$$

Definition 2.1. A strategy $\pi = (a^\pi; L^\pi; G^\pi)$ is said to be admissible if

(i) The ceded proportion $a^\pi = a_t^\pi$ is an $\mathcal{F}_t^B$-adapted process with $0 \le a_t^\pi \le 1$ for all $t \ge 0$.
(ii) $\{L_t^\pi\}$ is an increasing, $\mathcal{F}_t^B$-adapted càdlàg process with $L_{0-}^\pi = 0$, and satisfies that $\Delta L_t^\pi = L_t^\pi - L_{t-}^\pi \le X_{t-}^\pi$ for all $t \ge 0$.
(iii) $\{\tau_n^\pi\}$ is a sequence of stopping times w.r.t. $\mathcal{F}_t^B$, and $0 \le \tau_1^\pi < \cdots < \tau_n^\pi < \cdots$, a.s..
(iv) $\eta_n^\pi \ge 0, n = 1, 2, \cdots$ is measurable w.r.t. $\mathcal{F}_{\tau_n^\pi}^B$.
(v) $\mathrm{P}\left(\lim_{n\to\infty} \tau_n^\pi < T\right) = 0, \forall T > 0.$

The class of admissible strategies is denoted by Π.

For each strategy $\pi \in \Pi$, the bankruptcy time of the controlled process X_t^π is defined as $\tau^\pi = \inf\{t : X_t^\pi < 0\}$, which is an $\mathcal{F}_t^B$-stopping time. Note that the bankruptcy time could be infinite.

Problem 2.1. We define the company's value by the performance function $V(x,\pi)$, which is the expected sum of discounted salvage and the discounted dividends less the expected discounted costs of capital injections until bankruptcy

$$V(x, \pi) = \mathrm{E}^x\left(\beta_1 \int_0^{\tau^\pi} e^{-\delta s}\mathrm{d}L_s^\pi - \sum_{n=1}^{\infty} e^{-\delta\tau_n^\pi}\left(\beta_2\eta_n^\pi + K\right)I_{\{\tau_n^\pi \le \tau^\pi\}} + Pe^{-\delta\tau^\pi}\right). \tag{2.5}$$

E^x denotes the expectation conditional on $X_0^\pi = x$, and $\delta > 0$ is the discount rate. We regard $P \ge 0$ as the salvage value of the insurer; for example, an insurer's brand name or agency network which might be of value to a potential buyer of the insurer. We assume that the shareholders need to pay $\beta_2\eta + K$ to meet the capital injection of η. $\beta_2 > 1$ measures the proportional costs, $K > 0$ is the fixed costs. Proportional costs on dividend transaction are taken into account through the value of β_1, with $0 < \beta_1 \le 1$ representing the net proportion of leakages from the surplus received by shareholders after transaction costs have been paid. We are interested in finding the value function

$$V(x) = \max_{\pi \in \Pi} V(x, \pi) \tag{2.6}$$

and the associated optimal strategy π^* such that $V(x) = V(x, \pi^*)$.

Remark 2.1. The case of $P < 0$ is out of consideration in this paper. Since the surplus can keep nonnegative by ceding the whole risk to the