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# Pricing foreign equity options with regime-switching

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ABSTRACT

In this paper, we investigate the valuation of two types of foreign equity options under a Markovian regime-switching mean-reversion lognormal model, where some key model parameters in the dynamics of the foreign equity price and the foreign exchange rate are modulated by a continuous-time, finite-state Markov chain. A fast Fourier transform (FFT) approach is applied to provide an efficient way to evaluate the option prices. Numerical analysis and empirical studies are provided to illustrate the practical implementation of the proposed pricing model.



## 1. Introduction

Due to recent technological advance and trade liberalization, the growth of globalization has been accelerated and the economic growth has been boosted unprecedentedly. In the global financial markets, foreign exchange risk arising from fluctuations in foreign exchange rate has received much attention, especially since the currency crises in emerging markets. To hedge and manage foreign exchange risk, both academic researchers and industry practitioners have proposed a variety of currency options. Partly attributed to globalization, many firms and households are massively involved in investment activities of foreign assets. There are two key sources of risk arising in investment on foreign assets, namely foreign exchange (FX) risk and asset's price risk. Effective management of these two sources of risk is the key to successes in foreign assets investments. Foreign equity options provide a possible way to manage or hedge both the FX risk and the equity price risk. According to the definition in Kwok and Wong (2000), "the currency-translated foreign equity options are contingent claims whose payoffs are determined by financial prices or indices denominated in one currency but the actual payouts are settled in another currency". As its name implies, the underlying asset of a foreign equity option is a foreign equity. There are two main tempting features of foreign equity options. Firstly, foreign equity options provide investors with a variety of flexible ways to deal with the multidimensional risks, mainly the foreign equity price fluctuation risk and the foreign exchange risk. There exist a variety of types of foreign equity options with different payoff functions. Seen from this aspect, foreign equity options could provide investors with more investment and risk management choices. Exchange-trade is the second advantage of foreign equity options, which means this kind of financial product enjoys a higher degree of liquidity. Furthermore, the regulations of clearinghouse help investors reduce or avoid some risks, such as counterparty risk.

Since the pricing model of foreign equity options needs to depict the joint dynamics of the exchange rate and foreign equity prices, there are some literature about the valuation of foreign equity options under different models. Early works usually consider the valuation of foreign equity options in the Black–Scholes framework. Kwok and Wong (2000) investigated the valuation of foreign equity options with path dependent features. Examples of pricing foreign equity options beyond the traditional BS framework include a multi-dimensional Lévy process to depict the dynamics of both the exchange rate and the foreign equity prices in Huang and Hung (2005). Xu et al. (2011a) considered the valuation of foreign equity option under a stochastic volatility model with double jumps. To incorporate the impacts of skewness and kurtosis on foreign equity option prices, the Gram–Charlier series expansion approach was adopted by Xu et al. (2011b).

It is known that certain vital features of financial time series cannot be depicted by the classical Black-Scholes models. Among the models extending the classical Black–Scholes model, the ability to incorporate

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structural changes in economic conditions makes regime-switching models one of the most practically useful models in financial econometrics. These changes, which may be attributed to changes in economic fundamentals or business cycles, represent an additional source of risk to which an additional amount of risk premium may be required to compensate. Furthermore, the risk brought by these changes can be hardly diversified since it is more likely to be regarded as a systematic risk. Since regime-switching models provide a natural and convenient choice to model the structural changes in economic conditions, especially due to financial crises, this class of models will enjoy more and more popularity. The seminal work of Hamilton (1989) popularized applications of regime-switching models in financial econometrics. Typically, the so-called "modulated by a Markov chain" means the model dynamics or parameters will change when the underlying Markov chain changes from one state to another. The states of the Markov chain represent the states of an economy. Since the last decade or so, there has been an interest on studying option valuation problems in regime-switching models (see Buffington and Elliott (2002), Elliott et al. (2005), Siu (2008), Yuen and Yang (2010), Shen et al. (2013), Shen and Siu (2013), etc.). Considering the increasingly changing foreign exchange market, there is a considerable interest to investigate the valuation of currency options under regime-switching models, including Bollen et al. (2000), Siu et al. (2008), Bo et al. (2010). Empirical studies in Bollen et al. (2000) verified that trading strategies under regime-switching models can gain higher profit and be more attractive to investors. That also indicates the potential practical value of regime-switching models.

However, relatively little attention has been given to pricing foreign equity options in the context of regime-switching models. In this paper, we investigate the valuation of foreign equity options under a Markovian regime-switching mean-reversion lognormal model, which extends the mean-reversion lognormal model for foreign exchange rate. More specifically, the model parameters, including the risk-free domestic interest rate, the volatility of the foreign equity, the mean-reversion level and the volatility of the foreign exchange rate, as well as the instantaneous correlation coefficient between the foreign equity and the exchange rate, are modulated by a continuous-time, finite-state, observable Markov chain. To apply the fast Fourier transform (FFT) approach to discretize the integral pricing formula, we need to first calculate the characteristic function of the logarithmic underlying equity price. For the valuation of the foreign equity option with strike price in the foreign currency ($FEO_F$ [1]), we first apply a measure change technique and use a version of the Bayes' rule to derive the conditional characteristic function of the logarithmic equity price under the new measure. For the valuation of the foreign equity option with strike price in the domestic currency ($FEO_D$), we calculate the characteristic function of the summation of the logarithmic foreign equity price and the logarithmic foreign exchange rate under the risk-neutral probability measure. Then, we derive the Fourier transform of the foreign equity option price in these two cases. To illustrate the pricing of foreign equity options, we provide a numerical analysis using the FFT method. Finally, an empirical application is provided, revealing that the regime-switching model outperforms the model with a single regime in terms of lower fitting errors and prediction errors. The main contributions of this paper are as follows. (1) We investigate the valuation of foreign equity options under a regime-switching mean-reversion lognormal model. The main feature of our model is that it combines the advantages of both regime-switching models and mean-reversion lognormal models. The mean reversion feature of foreign exchange rates has been well-documented. (Jorion and Sweeney (1996); Sweeney (2006)) Wong and Lau (2008); Wong and Lo (2009); Wong and Zhao (2010); Leung et al. (2013)). (2) By applying a measure change technique, the Fourier transform of the $FEO_F$ option price can be calculated more easily. Then, we adopt the FFT approach in Carr and Madan (1999) and Liu et al. (2006) to derive a pricing formulae for the foreign equity options.

The rest of the paper is organized as follows. The next section presents the model dynamics. In Section 3, we derive the pricing formulae of $FEO_F$ and $FEO_D$ under the Markovian, regime-switching, mean-reversion lognormal model, respectively. Section 4 presents numerical examples. An empirical application of our model is provided in Section 5. The final section concludes the paper.

## 2. The model dynamics

In this section, we consider a continuous-time economy with a finite time horizon $\mathcal{T} := [0, T]$, where $T < \infty$. Let $(\Omega, \mathcal{F}\mathcal{P})$ be a complete probability space. In the literature about foreign exchange rate modeling, it is customary to assume that $\mathcal{P}$ is a risk-neutral probability measure (See Wong and Lau (2008)). To describe the evolution of the state of an economy over time, we consider a continuous-time, $N$-state, observable Markov Chain $\mathbf{X} := \{\mathbf{X}(t)|t\in\mathcal{T}\}$. The $N$ different states of the chain may represent $N$ observable different states of an economy or different stages of a business cycle. Without loss of generality, using the convention in Elliott et al. (1994), we assume the chain $\mathbf{X}$ has a canonical state space $\mathcal{E} := \{\mathbf{e}_1,\mathbf{e}_2,\ldots,\mathbf{e}_N\} \subset \Re^N$, where the $j$-th component of $\mathrm{e}_i$ is the Kronecker delta $\delta_{ij}$, for each $i,j = 1,2,\ldots,N$. Let $\mathbf{Q} := [q_{ij}]_{i,j=1,2,\ldots,N}$ denote the generator or rate matrix of the chain $\mathbf{X}$ under $\mathcal{P}$, where $q_{ij}$ is the transition intensity of the chain $\mathbf{X}$ from state $\mathrm{e}_i$ to state $\mathrm{e}_j$. Then the following semimartingale dynamics for the chain $\mathbf{X}$ were obtained in Elliott et al. (1994):

$$\mathbf{X}(t) = \mathbf{X}(0) + \int_0^t \mathbf{Q}\mathbf{X}(s)ds + \mathbf{M}(t), t\in\mathcal{T}.$$

Here $\{\mathbf{M}(t)|t\in\mathcal{T}\}$ is an $\Re^N$-valued, $(\mathbb{F}^{\mathbf{X}}, \mathcal{P})$-martingale, where $\mathbb{F}^{\mathbf{X}}$ is the right-continuous, $\mathcal{P}$-complete, natural filtration generated by the chain $\mathbf{X}$.

We now specify the Markovian regime-switching models for the dynamics of the foreign equity and the foreign exchange rate. Let $S := \{S(t)|t\in\mathcal{T}\}$ and $Z := \{Z(t)|t\in\mathcal{T}\}$ denote the price process of the foreign equity and the logarithmic foreign exchange rate process respectively. Let $\mathbf{y}'$ be the transpose of a vector or a matrix $\mathbf{y}$, $\langle\cdot,\cdot\rangle$ be the scalar product in $\Re^N$, and $diag(\mathbf{y})$ be the diagonal matrix with diagonal elements being given by the components of the vector $\mathbf{y}$. For each $t\in\mathcal{T}$, let $r(t)$ and $\sigma(t)$ be the domestic, instantaneous continuously compounded, interest rate and the volatility of the equity at time $t$, respectively. We assume that $r(t)$ and $\sigma(t)$ are determined by the value $\mathbf{X}(t)$ of the chain at time $t$ as:

$$r(t) := \langle \mathbf{r}, \mathbf{X}(t)\rangle,$$
$$\sigma(t) := \langle \boldsymbol{\sigma}, \mathbf{X}(t)\rangle,$$

where $\mathbf{r} := (r_1,r_2,\ldots,r_N)' \in \Re^N$ with $r_i > 0$ and $\boldsymbol{\sigma} := (\sigma_1,\sigma_2,\ldots,\sigma_N)' \in \Re^N$ with $\sigma_i > 0$ for each $i = 1,2,\ldots,N$.

Let $\{\alpha(t)|t\in\mathcal{T}\}$ and $\{\gamma(t)|t\in\mathcal{T}\}$ be the mean-reversion level and volatility of the process $Z$. Again we suppose that

$$\alpha(t) := \langle \boldsymbol{\alpha}, \mathbf{X}(t)\rangle,$$
$$\gamma(t) := \langle \boldsymbol{\gamma}, \mathbf{X}(t)\rangle,$$

where $\boldsymbol{\alpha} := (\alpha_1,\alpha_2,\ldots,\alpha_N)' \in \Re^N$ and $\gamma := (\gamma_1,\gamma_2,\ldots,\gamma_N)' \in \Re^N$ with $\gamma_i > 0$, for each $i = 1,2,\ldots,N$. The parameter $\beta$, controlling the speed of mean reversion for the logarithmic foreign exchange rate process, is assumed to be a positive constant.

[1] Following the notation in Xu et al. (2011b), let $FEO_F$ and $FEO_D$ represent the foreign equity option with strike price in the foreign currency and the foreign equity option with strike price in the domestic currency, respectively.