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Dynamic sentiment asset pricing model

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ABSTRACT

Conventional wisdom suggests that the equilibrium stock price is not affected by investor sentiment, and the equilibrium price at an early time is higher than the one at a later time. In contrast to this wisdom, we present a dynamic asset pricing model with investor sentiment and we find that investor sentiment has a significant impact on the equilibrium stock price. The equilibrium stock price, which is affected by pessimistic sentiment at time 0, may be lower than the one at time 1. Moreover, consistent with the reality stock market, our model shows that time varying sentiments can lead to various price changes. Finally, the model could offer a partial explanation for the financial anomaly of high volatility.

1. Introduction

Traditional asset pricing theory suggests that economists can safely ignore individual irrational behavior at the aggregate level (Friedman, 1953). But this argument is not adopted by behavioral finance, which argues that the investment strategy may be impacted by investor noise, investor psychology, investor sentiment and the arbitrage is limited. At first some noise trader models are proposed to illustrate the influence of noise on the stock price (De long et al., 1990; Yan, 2010). For example, Yan (2010) presented a noise model, where individual biases often cannot be canceled out by aggregation. The shortcoming of noise models is that the noise information is difficult to be identified and cannot be measured, consequently can't be empirically testified. Yang and Yan (2011) suggested that investor sentiment is easy to be measured by variant methods and the related result is supported by some financial experiments (Statman et al., 2008).

Nowadays, the systematic role of investor sentiment has been supported by some empirical and theoretical studies. A number of empirical studies have shown that investor sentiment has a systematic impact on stock return (Baker and Wurgler, 2006, 2007; Baker et al., 2012; Brown and Cliff, 2004; Kumar and Lee, 2006; Kurov, 2010; Lee et al., 1991; Liao et al., 2011; Schmeling, 2009; Verma and Soydemir, 2009; Yang and Zhang, 2013; Yu and Yuan, 2011).

Some static asset pricing models have been developed to support the role of investor sentiment, such as Yang et al. (2012), Yang and Yan (2011), Yang and Zhang (2013). Yang and Yan (2011) showed that the excess return is negatively related to a high sentiment bigger than a critical point, but positively related to a high sentiment smaller than this point. Yang et al. (2012) constructed a sentiment capital asset pricing model, and showed that investor sentiment is a nonlinear systematic factor for asset pricing. Yang and Zhang (2013) constructed a sentiment asset pricing model with consumption, and showed that the stock price has a wealth-weighted average structure and the investor's wealth proportion could amplify the sentiment shock on the asset price.

These static asset pricing models have showed the systematic impact of the investor sentiment on the stock price. However, compared with only one transaction, investors frequently trade stocks in capital market. Moreover, the static models usually assumed that the investor sentiment is a constant. In fact, the investors usually update their sentiment upon receiving more data. For example, the magnitude of investor sentiment decreases over time when investors are learning over time. Hence, the dynamic characteristic of stock price model is closer to the reality capital market and the setting of dynamic sentiment asset pricing model could explain the complex of the stock price changing form.

Based on the framework of a consumption-based model, we construct a dynamic asset pricing model with sentimental investors. Similar to the static model of Yang and Zhang (2013), the dynamic sentiment asset pricing model shows that investor sentiment has a significant impact on the equilibrium stock price. The equilibrium price is amplified by optimistic sentiment, and reduced by pessimistic sentiment. Conventional wisdom suggests that the equilibrium stock price at an early time is generally higher than the one at a later time. In contrast to this wisdom, our model shows that the stock price, which is affected by

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pessimistic sentiment at an early time, may be lower than the stock price at a later time. Our model could offer a partial explanation for the financial anomaly of high volatility.

One notable feature of Yan's model is that investors' beliefs do not change over time. This assumption is made for simplicity. In order to study the dynamic effect of time varying sentiment, this assumption is relaxed in our model. Our model shows that the equilibrium stock price at time 0 is impacted by time varying sentiments.

First, if the representative investor updates his sentiment upon receiving more information, then the magnitude of the investor's error decreases over time. Second, if the representative investor is pessimistic (or optimistic) at time 0 but optimistic (or pessimistic) at time 1, then the impact of pessimistic sentiment can be partly offset by the impact of optimistic sentiment. Finally, time varying sentiment can lead to a more significant price change.

The rest of this paper is organized as follows. Section 2 presents a dynamic model with investor sentiment. Section 3 presents the equilibrium stock price. Section 4 concludes.

2. The dynamic model

2.1. The economy

Our goal is to set up a dynamic model with investor sentiment to formalize the discussions outlined in the introduction. In this paper, we adopt the framework of consumption-based asset pricing model. One advantage of this dynamic model is that it is a general analytical framework including CAPM, APT and MPT.

Consider a two-period (three dates) model with $t = 0,1,2$. There are two tradable assets in the economy: a riskless bond and a stock. Given that the bond is assumed to be in perfectly elastic supply, and the interest rate r_f is set exogenously. The stock, normalized to one share, is a claim to two positive dividends D_1 and D_2. For $t = 1,2$,

$$\ln D_t = \ln D_{t-1} + g_t^D,$$

where g_1^D and g_2^D are independent and identically distributed (i.i.d.) with a distribution $N(\mu_D, \sigma_D^2)$ and $D_0 = 1$.

The exogenous aggregate consumption supply at time t is given by C_t, with $C_0 > 0$, and for $t = 1,2$,

$$\ln C_t = \ln C_{t-1} + g_t^c$$

where g_1^c and g_2^c are identically distributed with a distribution $N(\mu_c, \sigma_c^2)$. For simplicity, the shocks to dividends g_t^D and the shocks to consumption g_t^c are assumed to be independent.

2.2. Investor sentiment

That is an economy with a single consumer, interpreted as a representative stand in for many identical consumers at time 0, no share of stock and no bond. The systematic role of investor sentiment on stock returns has been proposed by some empirical and theoretical literatures (Baker and Wurgler, 2006; Brown and Cliff, 2004; Kumar and Lee, 2006). For example, Baker and Wurgler (2006) employed principal component analysis to construct a composite market sentiment index. They selected six variables, such as closed-end fund discount, turnover rate, number of IPO, first-day return of IPO, equity share in new issues, and dividends, as the proxies of market aggregate sentiment. They argued that this method can include more information of stock return. We now study the case in which consumers are affected by the investor sentiment.

In the market, investors would perceive stock dividend with some sentiment. For example, sentimental investors overestimate the stock dividend, and other investors underestimate the stock dividend. A sentimental investor is assumed to be pessimistic (or optimistic) about the dividend growth rate g_t^D. Suppose that investor i is pessimistic (or optimistic) about the mean growth rate:

$$\mu_D^i = \mu_D + f(SI), \tag{1}$$

where μ_D^i is a function of growth rate, SI is investor sentiment and $f(\cdot)$ is a sentiment function. Eq. (1) means that $g_1^D(SI)$ and $g_2^D(SI)$ are independent and identically distributed (i.i.d.) with a distribution $N(\mu_D^i, \sigma_D^2)$ in the sentiment investor's opinion.

Investor sentiment SI is an independent realization from S, which is a random variable and is distributed normally with mean $\bar{S}$ and variance σ_S^2. The random variable S is independent of all fundamentals. If investor i is a rational then SI is equal to 0. In general, the mean $\bar{S}$ is not equal to zero according to the results of empirical and financial experiments researches (Baker and Wurgler, 2006; Kumar and Lee, 2006; Statman et al., 2008). The mean sentiment $\bar{S}$ is a measure of the average "bullishness" of the sentiment trader. For example, $\bar{S}>0$ corresponds to the case in which investor sentiment is optimistic by aggregation in the bull market, and $\bar{S}<0$ corresponds to the case in which investor sentiment is pessimistic by aggregation in the bear market.

The sentiment function $f(\cdot)$ is the excess growth rate about the mean growth rate, and satisfies these properties as follows: (1) $f(\cdot) > 0$ when investor sentiment is optimistic ($SI > 0$); (2) $f(\cdot) < 0$ when investor sentiment is pessimistic ($SI < 0$); (3) $f(\cdot) = 0$ when investor sentiment is rational ($SI = 0$).

2.3. The optimization consumption problem

Assuming investor has the logarithm preference, which means that the utility function $U(\cdot)$ is $\ln(\cdot)$. Investor i's optimization consumption problem is

$$\max E_0^i\left[\sum_{t=0}^{2}\beta^t \ln(C_t)\right], 0<\beta<1,$$

where β is the patience parameter, and C_t is the consumption of sentiment investor i at time t. Budget constraint can be written as

$$W_{t+1}^i = r_{t+1}^s\left(W_t^i - C_t\right), \quad t = 0, 1,$$

where W_t^i is the investor i's wealth at time t, and r_{t+1}^s is the sentiment stock return at time $t + 1$. Denote the equilibrium stock price at time t as P_t. The sentiment return r_t^s is given by

$$r_1^s = \ln((D_1 + P_1)/P_0) = \ln\left(\left(D_0 \times e^{g_1^D(SI)} + P_1\right)/P_0\right)$$

$$r_2^s = \ln(D_2/P_1) = \ln\left(D_1 e^{g_2^D(SI)}/P_1\right),$$

where $E(g_1^D(SI)) = E(g_2^D(SI)) = \mu_D + f(SI)$.

The definition of a competitive equilibrium is standard: in the equilibrium, the sentiment investor maximizes the objective function and the good and financial markets clear. The standard consumption-based asset pricing model implies that the stock prices are given by the rational Euler equation:

$$1 = E\left(\beta(C_{t+1}/C_t)^{-1} r_t\right).$$

By a similar argument, the sentiment Euler equation is

$$1 = E\left(\beta(C_{t+1}/C_t)^{-1} r_t^s\right). \tag{2}$$

Compared with the rational Euler equation, the rational Euler equation is a special case of Eq. (2) when investor sentiment is zero. Eq. (2) shows that if the expected return is high when investor sentiment is