Economic Modelling 35 (2013) 45–50

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

Testing volatility persistence on Markov switching stochastic volatility models☆

Qi Pan, Yong Li *

Hanqing Advanced Institute of Economics and Finance, Renmin University of China, Beijing 100872, China

ARTICLE INFO

Article history:
Accepted 18 June 2013

JEL Classification:
C11
C12
C13

Keywords:
Bayes factor
Markov switching
Persistence
Stochastic volatility
Unit root

ABSTRACT

In the literature, some researchers found that the high persistence of the volatility can be caused by Markov regime switching. This concern can be reflected as a unit root problem on the basis of Markov switching models. In this paper, our main purpose is to provide a Bayesian unit root testing approach for Markov switching stochastic volatility (MSSV) models. We illustrate the developed approach using S&P 500 daily return covering the subprime crisis started in 2008.

1. Introduction

Over the past two decades, as an alternative to the GARCH model, the stochastic volatility (SV) model, introduced by Taylor (1986), is a powerful tool to capture the time-varying volatility. In option pricing, one limitation of the conventional Black–Scholes model is the constant assumption of the volatility. The traders have to change the assumption of the volatility in order to match the market prices. Meanwhile, the stochastic volatility setting can capture the clustering pattern observed in the market. The most famous paper by Hull and White (1987) used continuous time SV model in option pricing. And Shephard (2005) provided a detailed review of the development of this model in finance. The SV models are also brought to the dynamic stochastic general equilibrium (DSGE) framework in macroeconomics. Obviously, it is not convincing to assume that the volatility of the shock in the economy is constant. Fernandez-Villaverde and Rubio-Ramirez (2007) and Justiniano and Primiceri (2008) extended the DSGE models with stochastic volatility.

In the empirical literature, researchers often discovered that, in the real economy, the volatility was highly persistent. However, other stylized facts, such as the structural change, or shift in the mean of the volatility, may cause the persistence of the volatility moving towards one, see Lamoureux and Lastrapes (1990) and Hamilton and Susmel (1994). In order to obtain the consistent persistence, So et al. (1998) introduced regime-switching, proposed by Hamilton (1989), to the conventional SV model. They found that the persistence of the volatility would significantly drop. Actually, it is quite reasonable to allow the heterogeneity in the mean of the log-volatility. For instance, the so-called 'bear' and 'bull' markets in finance as well as the booming period and recession in macro-level economy can be possibly explained by the regime switching in the volatility. Although there are literatures, such as So et al. (1998) and Hwang et al. (2007), to argue that the persistence will drop if regime-switching is modeled, we still cannot completely confirm whether the close-to-unit-root volatility in pervious SV model is really caused by the regime switching.

In fact, if we compare the graph of these two models, it is difficult to discern the Markov-switching SV (MSSV) and the basic SV with high persistence, since both of them show similar clustering pattern. In this paper, our main idea is to incorporate regime switching into modeling, then check whether the volatility is still highly persistent. According to So and Li (1999), checking the volatility persistency can be formulated as unit root testing problem on the volatility models. So and Li (1999) first suggested using Bayes factor for testing the unit root in the SV model where the marginal likelihood method of Chib (1995) was used to estimate Bayes factor. Li and Yu (2010) pointed out that the method introduced by So and Li (1999) maybe numerically unreliable. Hence, Li and Yu (2010) showed that the Bayes factor for testing the unit root in the SV model can be written as the expectation of the ratio of un-normalized posteriors with respect to the posterior under the stationary stochastic volatility model. This idea was followed by

☆ This research is supported by Chinese National Science Fund under Nos.70901077 and 71271221.

* Corresponding author.
E-mail address: s04085590@gmail.com (Y. Li).

0264-9993/$ – see front matter. Crown Copyright © 2013 Published by Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.econmod.2013.06.029

Zhang et al. (2013), Li et al. (2012) for developing Bayesian unit root testing in the presence of possible stationary and nonstationary SV models for random shocks. In our paper, we utilize the same idea to check the persistence of the volatility in the Markov switching SV model.

The rest of this paper is organized as follows. Section 2 is the brief review of the estimation of SV and Markov switching SV models. In Section 3, we detail how to compute the Bayes factor for the unit root test on Markov switching SV model. In Section 4, we apply the Bayesian MCMC method to S&P 500 data covering subprime crisis and obtain the result of the Bayesian unit root test. At last, we conclude in Section 5.

2. The model

We first consider the basic stochastic volatility model given as follows:

$$\begin{aligned} y_t &= \exp(h_t/2)u_t \\ h_{t+1} &= \alpha + \phi(h_t - \alpha) + \sigma_\eta \eta_t \\ h_1 &\sim N\left(\alpha, {\sigma_\eta}^2 / \left(1-\phi^2\right)\right), \end{aligned} \tag{1}$$

where y_t is the rate of return of the stock or the growth rate of the macroeconomic data. h_t is the log volatility, and it captures the heterogeneity of the rate of return. Eq. (1) is the motion of the log volatility. And u_t and η_t are independent standard normal random shocks.

Adopting the idea in So et al. (1998), we add Markov-switching part to the unconditional mean of the log volatilityα, then the equation of h_t can be written as:

$$\begin{aligned} h_{t+1} &= \alpha_{s_{t+1}} + \phi\left(h_t - \alpha_{s_t}\right) + \sigma_\eta \eta_t \\ \alpha_{s_{t+1}} &= \gamma_1 + \gamma_2 I_{s_{t+1}\geq 2} + \ldots + \gamma_K I_{s_{t+1}\geq K} \\ h_1 &\sim N\left(\alpha_{s_1}, {\sigma_\eta}^2 / \left(1-\phi^2\right)\right), \end{aligned} \tag{2}$$

where s_t is the latent state with support {1,2,…,K} and $I_{s_{t+1}\geq i}$ is the indicator function that attains 1 if $s_{t+1} \geq i$, otherwise attains 0. Contrasted with the finite mixture model, in the Markov-switching model, the latent state is governed by a first order Markov chain. The transition matrix is P with $P_i\cdot = (p_{i1},\ldots,p_{iK})$ and $p_{ij} = p(s_t = j|s_{t-1} = i)$. Under this setting, the smaller the α, the less the volatility y_t will have. Here we only look into the case where K = 2. For instance, if γ_2 is larger than 0, than $s_t = 1$ indicates that the time series are with low volatilities and $s_t = 2$ implies that the volatilities are high.

2.1. Model estimation

Unlike the GARCH model, the SV model is difficult to be estimated via the conventional maximum likelihood method, since we cannot write down the likelihood function analytically. Harvey et al. (1994) employed the quasi-ML method to SV model. Kim et al. (1998) proposed Markov chain Monte Carlo method to sample from the posterior of the parameters. Following similar scheme, we can easily estimate the MSSV model via Bayesian MCMC method, only by adding an additional block to the procedure. Albert and Chib (1993) proposed a discrete filter to obtain the hidden state that governed the shift of the mean. Carter and Kohn (1994) and Chib (1996) also developed a discrete filter which is a multi-move sampler.

In Markov-switching setting, the mean of the log-volatility is driven by the hidden state. The identification of the state-specific parameters is a problem. Fruhwirth-Schnatter (2001) suggested using the permutation sampler to identify the state-depended parameters. Following their idea, the permutation sampler is used in this paper. For Markov switching SV models, if single-move method is used in Gibbs sampler, sampling the log-volatility from the full conditional distribution $\pi(h_t|h_{-t},\gamma_1,\gamma_2,\phi,\sigma\eta,s,y)$ (y and s here represent the entire sets of observed data and hidden states), where h_{-t} means all other log-volatility except h_t, is nontrivial because the single-move method generally adds additional blocks, which will lower down the convergence speed. In our paper, we adopt the simulation smoother proposed by De Jong and Shephard (1995), and plug it into the mixture model, which was suggested by Kim et al. (1998) to approximate the SV model using seven normal distributions. The ideal behind the simulation smoother is simple, we do not sample the log-volatility directly, instead, the η_t — disturbance of the motion of the log-volatility — is drawn.

After the log-volatility is sampled from its posterior, we are able to simulate the hidden state from the full conditional distribution $\pi(s|\gamma_1,\gamma_2,\phi,\sigma_\eta,h,P)$ (h here represents the whole set of log-volatilities). Albert and Chib (1993) developed a discrete filter that allows us to sample from $p(s_t|s_{-t},h,\theta,P)$ with $\theta = \{\gamma_1,\gamma_2,\phi,\sigma_\eta\}$. Carvalho and Lopes (2007) utilized the auxiliary particle filter introduced by Pitt and Shephard (1999) to sample the log-volatility and the hidden state simultaneously. Here, we modify the discrete filter in Chib (1996) to fit the parameterization form of our model. Let (sample size N):

$$\begin{aligned} S_t &= (s_1,\ldots,s_t), S^t = (s_t,\ldots,s_N) \\ H_t &= (h_1,\ldots,h_t), H^t = (h_t,\ldots,h_N). \end{aligned}$$

Under these notations, the first column is the history of h and s, the second one is the future information. The current log-volatility h_t not only depends on h_{t-1} and s_t, but also the lag hidden state s_{t-1}. The posterior $\pi(s|\theta,h,P)$can be written in the following way (we suppress θ and P for convenience):

$$p(S_N|H_N) = p\left(s_1\left|S^2, H_N\right.\right) \times p\left(s_2\left|S^3, H_N\right.\right) \times \ldots \times p(s_N|H_N). \tag{3}$$

Since s_{t-1} will enter into h_t, the posterior of the hidden state is a little bit different from that in Chib (1996). As to the other parts of full-conditional distributions, the sampling approaches provided in Kim et al. (1998) are followed. Appendix A collects all details about Gibbs sampler.

2.2. The prior

In this section, we list the prior distributions of the parameters used in this paper. We know that in Markov switching SV model one should choose the prior of the parameters very carefully, because the improper prior may lead to improper posterior. In So et al. (1998), they used non-informative prior for the state-dependent parameters, yet restriction is imposed on the hidden states — the state could not be degenerated. According to Fruhwirth-Schnatter (2005), if the model is indeed over-fitted, then the identification constraint cannot rule out the label switching problem. A simple way for avoiding this problem is to bound the prior away from the non-identifiable set. Moreover, it is known that when calculating the Bayes factor, the result is even more sensitive to the prior than that in the estimation. If we use improper prior in the calculation of Bayes factor, we may

Table 1
Summary of the S&P 500 data.

	Mean	S.D.	Skewness	Kurtosis
y_t	−3.6620e−04	0.0149	−0.3304	15.5534
$\log(y_t^2)$	−10.7834	2.5703	−0.8432	5.3010

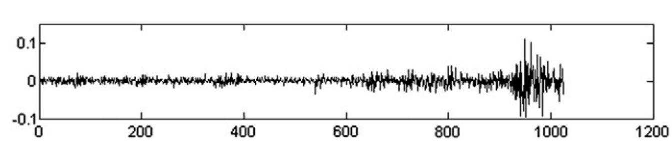

Fig. 1. Daily return for S&P 500 from Jan. 2, 2005 to Jan. 30, 2009.