Economic Modelling 35 (2013) 199–206

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

Analyzing the dependence structure of various sectors in the Brazilian market: A Pair Copula Construction approach

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ARTICLE INFO

Article history:
Accepted 5 June 2013

JEL classification:
C00
C5
G2

Keywords:
Dependence
Risk management
Pair Copula Construction
Sectors
Brazilian market

ABSTRACT

In this paper we estimate the dependence structure between economic sectors in the Brazilian financial market through Pair Copula Construction. We use daily data from indices which represent telecommunications, energy, industrials, consumer, financial, basic materials and real estate sectors in BM&F/Bovespa. Results indicate predominance of student's *t* copula in structure. BB1, BB7, BB8, Frank and Joe copulas also fit into some relationships. Regarding dependence, tail measures obtain relevant values in most relationships. Lower tail dependence exceeds absolute, measured by Kendall's Tau, and upper tail in many cases, reflecting the asymmetry in some relationships. Further, in order to give robustness to these results, we forecast daily Value at Risk, considering distinct significance levels, of a portfolio composed of studied sectors through the estimated structure. Results allow one to conclude that VaR predictions are correct. These results permit business industry participants to construct portfolios with assets of these sectors under a proper diversification structure. Moreover, from an international point of view, investors who are interested in diversification could perform more sophisticated strategies in this country rather than simply trading the index.

1. Introduction

Since the introduction of portfolio selection mathematical theory and Capital Asset Pricing Model (CAPM), the issue of dependence has been of fundamental importance to financial economics. In the context of international diversification, there is a need of minimizing risk of specific assets through optimal resource allocation. Therefore, it is necessary to understand multivariate relationships between different markets. Thus, we need a statistical model able to measure temporal dependence between shocks of different assets.

Corroborating with the increasing importance of financial asset dependence, emerges the fact that the extent of extreme shocks and potential damaging consequences continuously attract attention among economists and policymakers. These extreme dependencies, which represent probability function tails, beyond any fundamental link, for a while have been an issue of interest to academics, fund managers and traders, as it has important implications for portfolio allocation and asset pricing.

An inappropriate model for dependence can lead to suboptimal portfolios and inaccurate assessments of risk exposures. Traditionally, correlation is used to describe dependence between random variables, but recent studies have ascertained the superiority of copulas to model dependence, as they offer much more flexibility than the correlation approach, because a copula function can deal with non-linearity, asymmetry, serial dependence and also the well-known heavy-tails of marginal and joint probability distribution of financial assets.

A copula is a function that links univariate marginals to their multivariate distribution. Since it is always possible to map any vector of random variables into a vector with uniform margins, we are able to split margins of that vector and a digest of dependence, which is the copula. A great part of the research on copulas is still limited to the bivariate case. Thus, to construct higher dimensional copulas is the natural next step, despite its hardship. Apart from the multivariate Gaussian and Student, selection of higher-dimensional parametric copulas is still rather limited (Genest et al., 2009).

Developments in this area tend to hierarchical, copula-based structures. It is very possible that the most promising of these is the Pair Copula Construction (PCC). Originally proposed by Joe (1996), it has been further discussed and explored in the literature for questions of inference and simulation. PCC is based on a decomposition of a multivariate density into bivariate copula densities, of which some are dependence structures of unconditional bivariate distributions, and the rest are dependence structures of conditional bivariate distributions.

Regarding practical issues, a fundamental concern in financial markets is diversification of investments in distinct economic sectors. In that sense, dependence of shocks from one sector to another was

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http://dx.doi.org/10.1016/j.econmod.2013.06.012

documented by Ewing (2002), among others. The finding of these spillovers brings a whole new set of implications. Additionally, since different financial assets are traded based on these sector indices, it is important for financial market participants to understand the transmission across sectors in order to make an optimal portfolio allocation decision (Hassan and Malik, 2007).

Based on this perspective, this paper aims to estimate the dependence structure between economic sectors in the Brazilian financial market through PCC. For that, we collected daily data from BM&F/Bovespa indices which represent the telecommunications, energy, industrials, consumer, financial, basic materials and real estate sectors. The structure estimated allows one to calculate the tail dependence of each bivariate relationship between sectors, which gives investors relevant complementary information. Further, in order to give robustness to these estimates, we predicted the daily Value at Risk (VaR) of a portfolio composed of studied sectors.

The main contribution of this paper is to obtain information about the dependence of the economic sectors in the Brazilian market in order to aid market participants to construct portfolios with assets of these sectors under a proper diversification structure. Moreover, from an international point of view, Brazil has become an important alternative for diversification, especially since the flight of capital occasioned by recent financial crises, it is crucial for international investors to have a mapped scenario of the internal market of this country. This would allow international investors in Brazil to have more sophisticated strategies rather than simply negotiating the index.

The sequence of this paper is structured in the following way: Section 2 briefly exposes the background about copulas and PCC; Section 3 presents materials and methods, exposing data and procedures to achieve the paper objective; Section 4 presents results and their discussion; Section 5 concludes the paper.

2. Background

This section is subdivided on: i) Copulas, which briefly explains about definition and properties of this class of function; ii) Pair Copula Construction, which succinctly exposes the concepts of this structure.

2.1. Copulas

Dependence between random variables can be modeled by copulas. A copula returns the joint probability of events as a function of marginal probabilities. This property makes copulas attractive, as univariate marginal behavior of random variables can be modeled separately from their dependence (Kojadinovic and Yan, 2010).

The concept of copula was introduced by Sklar (1959). However, only recently its applications have become clear. A detailed treatment of copulas as well as their relationship to concepts of dependence is given by Joe (1997) and Nelsen (2006). A review of the applications of copulas in finance can be found in Embrechts et al. (2003) and in Cherubini et al. (2004).

For ease of notation we restrict our attention to the bivariate case. The extensions to the n-dimensional case are straightforward. A function $C:[0,1]^2 \rightarrow [0,1]$ is a *copula* if, for $0 \le x \le 1$ and $x_1 \le x_2$, $y_1 \le y_2$, (x_1, y_1), $(x_2, y_2) \in [0,1]^2$, it fulfills the following properties:

$$C(x,1) = C(1,x) = x, C(x,0) = C(0,x) = 0. \tag{1}$$

$$C(x_2,y_2) - C(x_2,y_1) - C(x_1,y_2) + C(x_1,y_1) \ge 0. \tag{2}$$

Property Eq. (1) means uniformity of the margins, while Eq. (2), the *n-increasing property* means that $P(x_1 \le X \le x_2, y_1 \le Y \le y_2) \ge 0$ for (X, Y) with distribution function C.

In the seminal paper of Sklar (1959), it was demonstrated that a Copula is linked with a distribution function and its marginal distributions. This important theorem states that:

(i) Let C be a copula and F_1 and F_2 univariate distribution functions. Then Eq. (3) defines a distribution function F with marginals F_1 and F_2.

$$F(x,y) = C(F_1(x), F_2(y)), (x,y) \in R^2. \tag{3}$$

(ii) For a two-dimensional distribution function F with marginals F_1 and F_2, there exists a copula C satisfying Eq. (3). This is unique if F_1 and F_2 are continuous and then, for every $(u,v) \in [0,1]^2$:

$$C(u,v) = F\left(F_1^{-1}(u), F_2^{-1}(v)\right). \tag{4}$$

In Eq. (4), F_1^{-1} and F_2^{-1} denote the generalized left continuous inverses of F_1 and F_2.

However, as Frees and Valdez (1998) note, it is not always obvious to identify the copula. Indeed, for many financial applications, the problem is not to use a given multivariate distribution but consists in finding a convenient distribution to describe some stylized facts, for example the relationships between different asset returns.

2.2. Pair Copula Construction

The PCC is a very flexible construction, which allows for free specification of $n(n-1)/2$ bivariate copulas. This construction was proposed by the seminal paper of Joe (1996), and it has been discussed in detail, especially, for applications in simulation and inference (Bedford and Cooke, 2001; Bedford and Cooke, 2002; Kurowicka and Cooke, 2006). The PCC is hierarchical in nature. The modeling scheme is based on a decomposition of a multivariate density into $n(n-1)/2$ bivariate copula densities, of which the first $n-1$ are dependence structures of unconditional bivariate distributions, and the rest are dependence structures of conditional bivariate distributions (Aas and Berg, 2011).

PCC is usually represented in terms of density. The two main types of PCC that have been proposed in the literature are C (canonical)-vines and D-vines. In this paper we focus on D-vine estimation, which accordingly to Aas et al. (2009) has the density as in formulation Eq. (5).

$$f(x_1,\cdots,x_n) = \prod_{k=1}^{n} f(x_k) \prod_{j=1}^{n-1} \prod_{i=i}^{n-j} c\left\{ \begin{array}{l} F\left(x_i | x_{i+1},\cdots,x_{i+j-1}\right), \\ F\left(x_{i+j} | x_{i+1},\cdots,x_{i+j-1}\right) \end{array} \right\}. \tag{5}$$

In Eq. (5), x_1, ⋯, x_n are variables; f is the density function; $c(\cdot,\cdot)$ is a bivariate copula density and the conditional distribution functions are computed, according to Joe (1996), by formulation Eq. (6).

$$F(x|\boldsymbol{v}) = \frac{\partial C_{x,v_j|\boldsymbol{v}_{-j}}\left\{F\left(x|\boldsymbol{v}_{-j}\right), F\left(v_j|\boldsymbol{v}_{-j}\right)\right\}}{\partial F\left(v_j|\boldsymbol{v}_{-j}\right)}. \tag{6}$$

In Eq. (6) $C_{x,v_j|\boldsymbol{v}_{-j}}$ is bivariate conditional distribution of x and v_j dependency structure conditioned on $\boldsymbol{v}_{-j}$, where the vector $\boldsymbol{v}_{-j}$ is the vector $\mathbf{v}$ excluding the component v_j. To become possible to use D-vine construction to represent a dependence structure through