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Kinetics of jobs in multi-link cities with migration-driven aggregation process

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ABSTRACT

Immigration has long been a hotly debated issue. The core of this debate is immigrants' impact on local job markets. Some people insist that instead of creating more jobs, immigrants actually take away more jobs thus decrease the living standard of natives. Others argue that the presence of immigrants benefits the society as a whole since they enlarge the labor force and lower the production cost. In this paper, we propose a model describing the migration-driven aggregation behaviors in job markets with foreign immigration, and introduce the method of network and aggregation to look at this issue from a new perspective. We divide the job market in each city into two groups: native and immigrant. And we view each city as a node with l links; each link represents a way of transportation to other cities. Then it is not hard to see that cities with more links tend to be more job concentrated with larger flows of jobs. We assume that both native and immigrant job markets have a migration of jobs within themselves and the native ones have birth rate and death rate of jobs as well. Through analyzing different rates: K_1 and K_2, initial conditions, and the combined effect of birth rate and death rate, we are able to predict the changes of some variables in the long run. These changes indicate the impact of immigrants on native job markets. Thus provide some helpful information to this issue.

1. Introduction

The aggregation process is of great significance in real life and in fields like physics (Cho and Kahng, 2011), biology (Capello et al., 2011) and economics (Horst and Scheinkman, 2009). Normally, we study the interaction among individual objects, but the aggregation process enables us to look at the interaction among groups (Moussaid et al., 2010) or individual object's behavior under group interaction (Ke and Lin, 2002). With the development of knowledge in network (Ke et al., 2006a, 2006b), we are capable of connecting aggregation process with social sciences (Bellaiche, 2010; Gonzalez-Avella et al., 2011). One important and popular application of aggregation process in social science is population dynamics (Brito and Dilao, 2010; Eckstein et al., 2011; Feng et al., 2010; Gupta and Dutta, 2011; Konan, 2011; Wang et al., 2007; Ziesemer, 2010). Population dynamics involves issues like population migration (Brito and Dilao, 2010; Feng et al., 2010; Konan, 2011), population growth (Wang et al., 2007) and job mobility (Eckstein et al., 2011; Gupta and Dutta, 2011; Ziesemer, 2010). Much research has contributed to social networks (Bramoulle and Saint-Paul, 2010) where scholars have tried different models to mimic some parts of real life interaction of people. This process is extremely hard because of the heterogeneity of individuals, but discoveries about social networks has helped solve some real problems like vaccination (Coudeville et al., 2009). The fundamental idea of social network is to regard each individual as a node, then the interactions among them are considered as ties. The introduction of aggregation process modifies this fundamental idea so that not only individuals, but also groups of people can be considered as nodes and the interactions among different groups are the corresponding ties. Equipped with this modification, later research focuses on groups (Ofiteru et al., 2010). Many choose to study a controversial issue: immigration. Some people support immigration and claim that immigrants bring benefit to the society they are heading to. Others hold different opinions and suggest that governments should carry out stricter regulations on immigrants. From an aggregation process point of view, immigrants is a group trying to aggregate to another group that could be seen as a nation or just an area.

The aggregation process functions through migration of individual monomer: $A_k + A_l \xrightarrow{K(k;l)} A_{k-1} + A_{l+1} \quad (k \leq l)$ (Leyvraz and Redner, 2002). Here A_k is an aggregate A with size k and $K(k;l)$ is the rate of migration from A_k to A_l. This scheme shows that for each individual monomer, it moves from smaller aggregates to bigger ones. However, a more general reversible reaction process exists as well where each monomer moves from bigger aggregates to smaller ones and vice versa (Ke and Lin, 2002). It is well known that in job markets, migration is reversible as jobs move from small cities to big ones and vice versa. Motivated by the above analysis, in our paper, we present a new job model in multi-link cities with migration-driven aggregation process, and investigate the evolution of job markets of different sizes in multi-link cities. To the best of our knowledge, few literature works on this topic.

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This paper is organized as follows. In Section 2, we derive the jobs migration model in multi-link cities, and analyze kinetics of jobs in multi-link cities with migration-driven aggregation process. Finally, Section 3 presents the concluding remarks.

2. Model description and analysis

We divide the job markets into two groups: $A_{k,l}$ means a city with l links has a native job market of size k; $B_{m,l}$ is a city with l links attached to it and has an immigrant job market of size m. A job belongs to the immigrant job market if it is taken by an immigrant. Similarly, a job position hold by a native is a part of the native job market. There exists a migration of jobs among job markets of the same kind. This migration is reversible. In addition, various $A_{k,l}$ and $B_{m,l}$ interact with each other as well. The presence of immigrants in the job markets creates jobs and takes away them at the same time. We call this creation rate as birth rate and the loss rate as death rate respectively. The links attached to each city here denote transportation methods like railway, airline or highways. The more links one job market has, the more jobs migrate in and out of it.

First we assume that job markets have spatial homogeneity. Their differences are showed only in the sizes of the job markets and the numbers of links attached. $a_{k,l}(t)$ is the number of city $A_{k,l}$ at time t and $b_{m,l}(t)$ is the number of city $B_{m,l}$ at time t. Then we can mimic the migration of jobs as follows:

$$\begin{cases} \dfrac{da_{k,l}}{dt} = a_{k+1,l}\sum_{j=1}^{N}\sum_{i=1}^{\infty}K_1(k+1;l|i;j)a_{i,j} + a_{k-1,l}\sum_{j=1}^{N}\sum_{i=1}^{\infty}K_1(i;j|k-1;l)a_{i,j} \\ \qquad -a_{k,l}\sum_{j=1}^{N}\sum_{i=1}^{\infty}[K_1(k;l|i;j)+K_1(i;j|k;l)]a_{i,j} \\ \qquad -a_{k,l}\sum_{j=1}^{N}\sum_{i=1}^{\infty}I(k;l|i;j)b_{i,j} + a_{k-1,l}\sum_{j=1}^{N}\sum_{i=1}^{\infty}I(k-1;l|i;j)b_{i,j} \\ \qquad +a_{k+1,l}\sum_{j=1}^{N}\sum_{i=1}^{\infty}J(k+1;l|i;j)b_{i,j} - a_{k,l}\sum_{j=1}^{N}\sum_{i=1}^{\infty}J(k;l|i;j)b_{i,j}, \\ \dfrac{db_{m,l}}{dt} = b_{m+1,l}\sum_{j=1}^{N}\sum_{i=1}^{\infty}K_2(m+1;l|i;j)b_{i,j} + b_{m-1,l}\sum_{j=1}^{N}\sum_{i=1}^{\infty}K_2(i;j|m-1;l)b_{i,j} \\ \qquad -b_{m,l}\sum_{j=1}^{N}\sum_{i=1}^{\infty}[K_2(m;l|i;j)+K_2(i;j|m;l)]b_{i,j}, l=1,2,\cdots,N. \end{cases} \tag{1}$$

Both natives and immigrants move around seeking new jobs. The migration rates among native job markets or immigrant ones are denoted as $K_1(k;\ l|i;\ j)$ and $K_2(k;\ l|i;\ j)$, respectively. $K_1(k;\ l|i;\ j)$ means the rate of one native job moves from $A_{k,l}$ to $A_{i,j}$: $A_{k,l}+A_{i,j}\xrightarrow{K_1(k;l|i;j)}A_{k-1,l}+A_{i+1,j}$. Similarly, $K_2(k;\ l|i;\ j)$ represents the rate of one immigrant job movement from $B_{k,l}$ to $B_{i,j}$. For the convenience of solving the rate equations, we work on typical symmetrical migration rates $K_1(k;\ l|i;\ j)=K_1ki$ and $K_2(m;\ l|i;\ j)=K_2mi$ which are proportional to the sizes of two corresponding job markets. Besides the migration of jobs within the same group, the flow of immigrant jobs has impacts on the natives ones. The birth reaction is shown by $I(k;\ l|i;\ j)=Iki^{\mu}$ as $A_{k,l}+B_{i,j}\xrightarrow{I(k;l|i;j)}A_{k+1,l}+B_{i,j}$. We call $I(k;\ l|i;\ j)$ as birth rate here, which means the migration of immigrants creates new jobs for natives at a certain rate. Compared to the birth rate, there is also a death rate $J(k;\ l|i;\ j)=Jki^{\mu}$ where immigrants cause the shrink of native job markets through the interaction $A_{k,l}+B_{i,j}\xrightarrow{J(k;l|i;j)}A_{k-1,l}+B_{i,j}$. These two rates are proportional to the size of the native job market that is affected. In addition, μ is the parameter showing the dependence of the birth or death effect on the size of the immigrant job market. The rate equations for our system (1) is reduced to

$$\begin{cases} \dfrac{da_{k,l}}{dt} = K_1M_1^A\left[(k+1)a_{k+1,l}+(k-1)a_{k-1,l}-2ka_{k,l}\right]+IM_\mu^B[(k-1)a_{k-1,l} \\ \qquad -ka_{k,l}]+JM_\mu^B\left[(k+1)a_{k+1,l}-ka_{k,l}\right], \\ \dfrac{db_{m,l}}{dt} = K_2M_1^B\left[(m+1)b_{m+1,l}+(m-1)b_{m-1,l}-2mb_{m,l}\right], l=1,2,\cdots,N. \end{cases} \tag{2}$$

where $M_\mu^A=\sum_{j=1}^{N}\sum_{k=1}^{\infty}k^\mu a_{k,j}(t)$ and $M_\mu^B=\sum_{j=1}^{N}\sum_{m=1}^{\infty}m^\mu b_{m,j}(t)$ are the μ-th moment of $a_{k,l}(t)$ and $b_{m,l}(t)$ with link number l, respectively. $M_0^A=\sum_{j=1}^{N}\sum_{k=1}^{\infty}a_{k,j}(t)$ and $M_1^A=\sum_{j=1}^{N}\sum_{k=1}^{\infty}ka_{k,j}(t)$ are the number of cities with a native job market and the sum of native jobs at time t, respectively. In this paper, we find that our current rate equations (2) can be solved by the Ansatz (Ke and Lin, 2002)

$$a_{k,l}(t)=A_l(t)[a_l(t)]^{k-1}, b_{m,l}(t)=B_l(t)[b_l(t)]^{m-1}. \tag{3}$$

Substituting Ansatz (3) into Eq. (2), we obtain the differential equations as follows:

$$\begin{cases} \dfrac{da_l}{dt} = \left[K_1M_1^A(1-a_l)+IM_\mu^B-JM_\mu^Ba_l\right](1-a_l), \\ \dfrac{dA_l}{dt} = -\left[2K_1M_1^A(1-a_l)+IM_\mu^B+JM_\mu^B-2JM_\mu^Ba_l\right]A_l, \\ \dfrac{db_l}{dt} = K_2M_1^B(1-b_l)^2, \\ \dfrac{dB_l}{dt} = -2B_lK_2M_1^B(1-b_l), l=1,2,\cdots,N. \end{cases} \tag{4}$$

The initial condition is

$$a_l(0)=0, A_l(0)=A_{0l}, b_l(0)=0, B_l(0)=B_{0l}, \text{at} \quad t=0. \tag{5}$$

From Eq. (4), we have $B_l(t)=B_{0l}[1-b_l(t)]^2$, this reveals the total number of immigrant jobs is conserved:

$$M_1^B=\sum_{j=1}^{N}\sum_{i=1}^{\infty}ib_{i,j}(t)=\sum_{j=1}^{N}\frac{B_j}{\left(1-b_j\right)^2}=\sum_{j=1}^{N}B_{0j}. \tag{6}$$

From Eq. (4)–(6), we obtain

$$b_l(t)=1-\left(1+K_2\sum_{j=1}^{N}B_{0j}t\right)^{-1}.$$

Hence,

$$\begin{cases} B_l(t)=B_{0l}\left(1+K_2\sum_{j=1}^{N}B_{0j}t\right)^{-2}, \\ b_{m,l}(t)=B_{0l}\left(1+K_2\sum_{j=1}^{N}B_{0j}t\right)^{-2}\left[1-\left(1+K_2\sum_{j=1}^{N}B_{0j}t\right)^{-1}\right]^{m-1}. \end{cases} \tag{7}$$

In the region of $t\gg1$ and $m\gg1$, Eq. (7) can be asymptotically rewritten as follows:

$$b_{m,l}(t)\cong B_{0l}\left(K_2\sum_{j=1}^{N}B_{0j}t\right)^{-2}exp\left\{-m\left(K_2\sum_{j=1}^{N}B_{0j}t\right)^{-1}\right\}. \tag{8}$$