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Risk index based models for portfolio adjusting problem with returns subject to experts' evaluations

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ABSTRACT

This paper discusses a portfolio adjusting problem with additional risk assets and a riskless asset in the situation where security returns are given by experts' evaluations rather than historical data. Uncertain variables are employed to describe the security returns. Using expected value and risk index as measurements of portfolio return and risk respectively, we propose two portfolio optimization models for an existing portfolio in two cases, taking minimum transaction lot, transaction cost, and lower and upper bound constraints into account. In one case the riskless asset can be both borrowed and lent freely, and in another case the riskless asset can only be lent and the borrowing of riskless asset is not allowed. The adjusting models are converted into their crisp equivalents, enabling the users to solve them with currently available programming solvers. For the sake of illustration, numerical examples in two cases are also provided. The results show that under the same predetermined maximum tolerable risk level the expected return of the optimal portfolio is smaller when the riskless asset can only be lent than when the riskless asset can be both borrowed and lent freely.

1. Introduction

Portfolio selection discusses allocation of one's capital to a number of securities to obtain the most profitable return with risk control. Since Markowitz (1952), portfolio theory has been developed greatly and a number of selection methods have been proposed. These methods can be classified into two basic types of methodologies. One type of the methodology is based on the risk measurements which directly gauge the deviation levels from the expected return but indirectly gauge the investors' loss of money. With this type, we have varieties of mean-variance models (DeMiguel et al., 2009; Kan and Smith, 2008), mean-semivariance models (Grootveld and Hallerbach, 1999; Markowitz et al., 1993), mean-expected absolute deviation (Konno and Yamazaki, 1991; Liu, 2011; Yu et al., 2010) models, etc. Since investors are usually more concerned about their loss of money, the second type of methodology is proposed based on the risk measurements which directly gauge the investors' losses. Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR, see Artzner et al., 1997) are representatives of these risk measurements. Since they make the loss recognizable, VaR and CVaR are becoming more and more popular in portfolio management. Varieties of VaR models (Berkowitz et al., 2011; Natarajan et al., 2008) and CVaR models (Hong and Liu, 2009; Lim et al., 2011; Zhu and Fukushima, 2009) have been studied. Recently, Huang (in press) proposed a risk index which directly gauges the average lost money below the riskless return rate as an alternative risk measurement, and added a mean-risk index model to the second type of methodology. In this paper, we will also use the risk index to directly measure the investors' loss of money and discuss an optimal portfolio adjusting problem. As we know, in practice investors may already have had a portfolio. Responding to changed situation in financial market, they often want to sell some holding securities and buy some new ones. Therefore, it is necessary to develop an adjusting model for an existing portfolio. Since there usually exist minimum transaction lot requirement and transaction costs in many financial markets, and institutions and individuals are often restrained by law or by their own choice from investing more than a certain percentage of their funds in any one security, we will take these practical constraints into account. Furthermore, in reality the investors can not only buy or sell risk assets but also lend or borrow riskless asset, therefore, we will consider the existence of riskless asset in the adjusting model.

In real life there are situations where there are no sufficient historical data (e.g. newly listed securities do not have enough historical data) or the historical data are not stable. Then security returns have to be given mainly by experts' estimations rather than historical data and thus contain much subjective imprecision rather than randomness. The purpose of this paper is to discuss the portfolio adjusting problem in this situation based on uncertainty theory.

The rest of the paper is organized as follows. In Section 2, we will briefly introduce the necessary knowledge about the uncertainty theory and review the measurement of risk index and compare it with the risk idea of CVaR. Then in Section 3 we will provide an optimal adjusting model considering minimum transaction lot, transaction costs and upper capital bound constraints. For the sake of illustration, we will

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present an example in Section 4. Finally in Section 5, we will give some concluding remarks.

2. Expected return and risk index based on uncertainty theory

To describe men's imprecise estimations of security returns, scholars have tried using fuzzy numbers, e.g. Huang (2008), Zhangb et al. (2011), Li et al. (2012), etc. However, further research has found that paradoxes will appear if we use fuzzy variable to describe the subjective estimations of security returns. Let us mention one example here. If a security return is regarded as a fuzzy variable, then we should have a membership function to characterize it. Suppose it is a triangular fuzzy variable $\xi=(-0.2, 0.2, 0.6)$. In fuzzy set theory, there are two basic measures, i.e., possibility measure and necessity measure. Based on the membership function, it is known from fuzzy set theory that the return is exactly 0.2 and not exactly 0.2 have the same possibility and necessity values, which implies that the two events will happen equally likely. This conclusion is too astonishing to accept. To better describe the subjective imprecise quantity, Liu (2007) proposed an uncertain measure and further developed an uncertainty theory in 2007 via an axiomatic system of normality, monotonicity, self-duality, countable subadditivity and product measure. With uncertainty theory, an uncertain variable was characterized by an uncertainty distribution. When we use uncertain variable to describe the human imprecise estimations of security returns, the above-mentioned paradox will disappear. Nowadays, scholars have successfully employed the uncertainty theory to handle many problems with imprecise information given by men. For example, Zhu (2010) solved an uncertain optimal control problem in portfolio selection. Huang proposed a mean-risk curve (Huang, 2011) and a mean-variance method (Huang, 2012) to handle portfolio optimization with returns given by experts' judgments. Other portfolio selection methods based on uncertainty theory can be found in Huang (2010). In addition, Gao (2011) solved a shortest path problem with arc lengths offered by experts' evaluations, and Zhanga et al. (2011) proposed a multi-national project selection method. Other applications of the uncertainty theory can be found in the areas of subjective uncertain logic (Chen and Ralescu, 2011, subjective uncertain inference (Gao et al., 2011), etc. In this paper, we use uncertain variables to describe the experts' estimations of security returns and uncertainty measure to gauge the occurrence chance of an uncertain event.

Definition 1. (Liu, 2007) Let Γ be a nonempty set, and $\mathcal{L}$ a σ-algebra over Γ. Each element $\Lambda\in\mathcal{L}$ is called an event. A set function $\mathcal{M}\{\Lambda\}$ is called an uncertain measure if it satisfies the following four axioms:

(Axiom 1) (normality) $\mathcal{M}\{\Gamma\}=1$.
(Axiom 2) (self-duality) $\mathcal{M}\{\Lambda\}+\mathcal{M}\{\Lambda^c\}=1$.
(Axiom 3) (countable subadditivity) For every countable sequence of events $\{\Lambda_i\}$, we have $\mathcal{M}\{U_{i=1}^{\infty}\}\le\sum_{i=1}^{\infty}\mathcal{M}\{\Lambda_i\}$.
(Axiom 4) (product measure) Let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ be uncertainty spaces for $k=1,2\cdots,n$. The product uncertain measure is $\mathcal{M}\{\Pi_{k=1}^{n}\Lambda_k\}=\underbrace{\min}_{1\le k\le n}\mathcal{M}_k\{\Lambda_k\}$.

The triplet $(\Gamma,\mathcal{L},\mathcal{M})$ is called an uncertainty space. It can be proven that the uncertainty measure is increasing. That is, $\mathcal{M}\{\Lambda_1\}\le\mathcal{M}\{\Lambda_2\}$ whenever $\Lambda_1\subset\Lambda_2$.

Definition 2. (Liu, 2007) An uncertain variable is defined as a measurable function ξ from an uncertainty space $(\Gamma,\mathcal{L},\mathcal{M})$ to the set of real numbers, i.e., for any Borel set of B of real numbers, the set

$$\{\xi\in B\}=\{\gamma\in\Gamma|\xi(\gamma)\in B\}$$

is an event.

In application, a random variable is usually characterized by a probability density function or probability distribution function. Similarly, an uncertain variable can be characterized by an uncertainty distribution function.

Definition 3. (Liu, 2007) The uncertainty distribution $\Phi:\Re\to[0,1]$ of an uncertain variable ξ is defined by

$$\Phi(t)=\mathcal{M}\{\xi\le t\}.$$

For example, an uncertain variable ξ is called the normal uncertain variable if it has the following distribution function

$$\Phi(t)=\left(1+exp\left(\frac{\pi(\mu-t)}{\sqrt{3}\sigma}\right)\right)^{-1}, t\in\Re \tag{1}$$

where μ and σ are real numbers and $\sigma>0$. For convenience, it is denoted by $\xi\sim\mathcal{N}(\mu,\sigma)$.

When the uncertain variables $\xi_1,\xi_2\cdots,\xi_n$ are represented by uncertainty distributions, the operational law is given by Liu (2010) as follows:

Theorem 1. (Liu, 2010) Let $\xi_1,\xi_2,\cdots\xi_n$ be independent uncertain variables with uncertainty distributions $\Phi_1,\Phi_2\cdots,\Phi_n$, respectively. Let $f(t_1,t_2,\cdots t_n)$ be strictly increasing with respect to $t_1,t_2,\cdots t_n$. Then

$$\xi=f(\xi_1,\xi_2,\cdots\xi_n)$$

is an uncertain variable whose inverse uncertainty distribution function is

$$\Psi^{-1}(\alpha)=f\left(\Phi_1^{-1}(\alpha),\Phi_2^{-1}(\alpha),\cdots,\Phi_n^{-1}(\alpha)\right) \tag{2}$$

if $\Phi_1^{-1}(\alpha)\Phi_2^{-1}(\alpha),\cdots\Phi_n^{-1}(\alpha)$ are unique for each $\alpha\in(0,1)$.

To tell the size of an uncertain variable, Liu defined the expected value of uncertain variables.

Definition 4. (Liu, 2007) Let ξ be an uncertain variable. Then the expected value of ξ is defined by

$$E[\xi]=\int_0^{\infty}\mathcal{M}\{\xi\ge t\}dt-\int_{-\infty}^{0}\mathcal{M}\{\xi\le t\}dt \tag{3}$$

provided that at least one of the two integrals is finite.

It can be calculated that the expected value of the normal uncertain variable $\xi\sim\mathcal{N}(\mu,\sigma)$ is $E[\xi]=\mu$.

Definition 5. (Huang, in press) Let ξ denotes an uncertain return rate of a risk asset, and r_f the riskless interest rate. Then the risk index of the asset is defined by

$$RI(\xi)=E\left[\left(r_f-\xi\right)^+\right], \tag{4}$$

where E is the expected value operator of the uncertain variable and

$$\left(r_f-\xi\right)^+=\begin{cases} r_f-\xi, & \text{if } \xi\le r_f\\ 0, & \text{if } \xi> r_f\end{cases}. \tag{5}$$

It is seen that the risk index is an average return level below the riskless interest rate r_f. It has some similarity to the risk measurement idea of CVaR in that they both are a kind of average loss value. However, in the framework of CVaR, the return rates below zero are regarded as losses, while in the risk index, the return rates above zero but below the riskless return rate are also regarded as losses. Since the riskless return rate is what investors can obtain with certainty, the definition of loss in the risk index is more reasonable than that in CVaR. In addition, in the framework of CVaR, the reference return ratio is VaR value, which is not known before the decision of portfolio selection; while in the