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A comparison of spatial error models through Monte Carlo experiments

Takafumi Kato *

Graduate School of Environmental Studies, Nagoya University, Nagoya, Aichi 464-8601, Japan

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ABSTRACT

A spatial error model is classified as a geostatistical model or a weight matrix model on the basis of the method of specification of spatial autocorrelation in the disturbance. Specification errors cannot be assumed to be absent, and the robustness of alternative specifications is useful for dealing with potential errors. Previous studies compared several models to arrive at two basic conclusions: (i) all of the models maintain reasonable estimation accuracy, and (ii) the two types of models have well-matched predictive abilities. The present study makes a supplementary comparison to investigate whether these conclusions are true for a broader range of models. Also, implications of our results for the model choice are explored.

1. Introduction

A linear regression model is called a spatial error model when spatial autocorrelation is present in the disturbance. A spatial error model is classified as a geostatistical model or a weight matrix model on the basis of the method of specification of spatial autocorrelation. The geostatistical model is defined with a correlation function, which is a device to specify the autocorrelation in a direct manner, whereas the weight matrix model is defined with a weight matrix, which is a device to specify it in an indirect manner. Unfortunately, errors cannot be assumed to be absent in any specification. As stated in Anselin (2002), even when the type of model is known, the choice of the correct correlation function or weight matrix is not theoretically guaranteed under any circumstances, and only a few incorrect choices can be practically eliminated using validation techniques. Thus, it is useful to explore the robustness of spatial autocorrelation specifications. Choosing a relatively well-performing model is a good policy to allow for specification errors.

In a recent study, Dubin (2003) took important steps toward exploring the robustness of alternative specifications. Under a reciprocal procedure, she performed a series of Monte Carlo experiments to compare three geostatistical and five weight matrix models, and found two basic conclusions: (i) all of the models maintain reasonable estimation accuracy, and (ii) the geostatistical models have better predictive abilities than the weight matrix models. Specifically, in each experiment, one of the models was used as the data generator, and the generated data were used to examine all of the models with conventional estimators and predictors.[1] In a following study, Kato (2008a) took further steps by showing that the conventional predictor applied to the geostatistical models is more efficient than that applied to the weight matrix models. He proposed two alternative predictors for the weight matrix models to make an optimal comparison of the two types of models, and tested the dominance of the geostatistical models over the weight matrix models.[2] As expected, in his experiments based on Dubin (2003), conclusion (i) was confirmed to be sound, but conclusion (ii) was shown to be unsound. In a comment on Kato (2008a), although appreciating his results, Dubin (2008) argued that the geostatistical models are still preferable to the weight matrix models. Two reasons were adduced in favor of this argument: (i) the difference in the predictive ability between the best and worst models is smaller for the geostatistical models than for the weight matrix models, and (ii) the predictive ability of the worst geostatistical model is greater than that of the worst weight matrix model. In a response to Dubin (2008), although acknowledging her observations, Kato (2008b) suggested a combined use of the geostatistical and weight matrix models that have the advantage in predictive ability among models of their respective type. Two reasons were produced in support of this suggestion: (i) all of the models offer the best prediction performance only in a limited number of

[1] Actually, Dubin (2003) did not use one weight matrix model as the data generator, but used it in estimation and prediction as a substitute for another weight matrix model that did not offer the best prediction performance in any experiment.

[2] Bourassa et al. (2007) compared the two types of models through an observed data experiment and concluded that a weight matrix model is not suitable for the purpose of prediction. Kato (2008a) showed that they applied to the weight matrix model a less efficient predictor than Dubin (2003).

* Tel.: +81 52 789 3502.
E-mail address: q47418a@cc.nagoya-u.ac.jp.

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experiments, and (ii) neither of the two types of models is dominant. Interestingly, the most advantageous weight matrix model is a model that mimics one of the geostatistical models.

The purpose of the present study is to make a supplementary comparison of spatial error models through a series of Monte Carlo experiments based on Dubin (2003). This comparison is meant to explore the robustness of spatial autocorrelation specifications in the following ways. First, we assess a weight matrix model that Kato (2008a) considered to be worth assessing in a future study. This model was proposed by Pace and Gilley (1997) to demonstrate the benefits from using a spatial error model. Second, we propose and assess two weight matrix models to consider the above argument of Dubin (2008). These models mimic two of the three geostatistical models examined in Dubin (2003) and Kato (2008a), respectively, in the same manner as one of the five weight matrix models examined there mimics the third geostatistical model. Observations similar to those made by Dubin (2008) for the geostatistical models could be made for the weight matrix models that mimic those models: the difference in the predictive ability between the best and worst models could be smaller for the three mimics than for a broader range of weight matrix models, and the predictive ability of the worst mimic could be greater than that of the worst weight matrix model. Third, we investigate whether we can choose the same combination of geostatistical and weight matrix models as Kato (2008b) when we consider the three additional models. Different models could be shown to be the most advantageous geostatistical and weight matrix models, and there is a possibility of finding an excellent model.

In forming our purpose, we have the hedonic price function in mind, because the spatial error model is finding wide application to that function. This is to follow Dubin (2003) and Kato (2008a), and is not to deny that the model is also applicable to other functions, as intimated in Arbia et al. (2012). If the term *spatial error model* is always used without relating it to any particular function, it is possible to make our comparison useful for these other functions. This usage is therefore adopted here. We also have reasons for taking up only a few weight matrix models as the additional targets of comparison. One reason is that the developments of the two types of models are different in the hedonic literature. As stated in Palmquist (2005), geostatistical models used are chiefly limited to the three above, whereas weight matrix models proposed are widely diverse.[3] Another reason is that it is not easy to put many models through the above reciprocal procedure. As the number of models compared increases, the amount of resources required jumps.

Naturally, the developments of the two types of models in the hedonic literature are reflected in the experiments of Dubin (2003) and Kato (2008a), and are influential in the discussions of Dubin (2008) and Kato (2008b). This indicates that the overall difference in the predictive ability found between the two types of models may be attributable to the difference between the diversities of models considered for their respective type. Unfortunately, the reason for this difference in the predictive ability was not explained in any of those previous studies. Assessment of the above three mimics allows us to discuss that hypothesis. If the overall performance of these weight matrix models is found to be similar to that of the three geostatistical models, the hypothesis may be accepted; otherwise, the difference in the predictive ability should be basically attributable to the difference in the method of specification of spatial autocorrelation.

The remainder of the present paper is organized as follows. In Section 2, we provide the aspects of comparison. Models are defined, and statistics are described. In Section 3, we perform Monte Carlo experiments. The design is produced, and the results are discussed. In Section 4, we summarize the points made in the preceding sections and mention topics for future study.

[3] Interestingly, Palmquist (2005) saw practical merit in the weight matrix model of Pace and Gilley (1997), which supports our assessment of that model.

2. Aspects of comparison

The mathematical notation of Kato (2008a) is adopted, with minor modifications, to provide the aspects of comparison. We first define the geostatistical and weight matrix models before proceeding to describe the estimators and predictors for the respective types of models.

2.1. Models

The spatial error model can be expressed as $\mathbf{y}=\mathbf{Xb}+\mathbf{u}$, where $\mathbf{y}$ is the vector of values of the dependent variable; $\mathbf{X}$ is the matrix of values of the independent variables, with the first column comprising ones; $\mathbf{b}$ is the vector of regression parameters b_1, b_2, $\cdots$, and b_m; and $\mathbf{u}$ is the vector of values of the disturbance.[4] In this model, $\mathbf{u}$ is assumed to have a normal distribution with expectation $\mathbf{0}$ and covariance $\sigma^2K(\mathbf{D};\mathbf{c})$, where σ^2 is a nuisance parameter, $\mathbf{c}$ is the vector of spatial autocorrelation parameters c_1 and c_2, $\mathbf{D}$ is a symmetric matrix of distances separating the locations included in the population, and $K(\circ;\mathbf{c})$ is a function in which the input and output are matrices of the same order. The model is classified as a geostatistical model or a weight matrix model on the basis of the method of specification of $K(\circ;\mathbf{c})$. In the geostatistical model, $K(\mathbf{D};\mathbf{c})$ is set to $[\ F(D_{ij};\mathbf{c})\ ]$, where D_{ij} expresses the ijth element of $\mathbf{D}$, which is the distance between the ith and jth locations, and $F(\circ;\mathbf{c})$ is a function in which the input and output are scalars.[5] This function is known as a correlation function, and its use indicates that the covariance between the disturbances associated with a pair of locations depends only on the separation distance of that pair. In the weight matrix model, $K(\mathbf{D};\mathbf{c})$ is set to $\left\{(\mathbf{I}-c_1W(\mathbf{D};c_2))'(\mathbf{I}-c_1W(\mathbf{D};c_2))\right\}^{-1}$, where $W(\circ;c_2)$ is a function in which the input and output are matrices of the same order. The output is known as a weight matrix, and its use indicates that the covariance between the disturbances associated with a pair of locations depends on the separation distances of all pairs.[6]

As an expedient for model definition, we assign the same names as Dubin (2003) and Kato (2008a) to their originally compared three geostatistical and five weight matrix models: *NEGE*, *GSS*, and *SPH* to the former models and *NN*, *K*, *P*, *LIM*, and *NEW* to the latter. Geostatistical models vary in accordance with the specification of the form of the correlation function, which determines the elements of $K(\mathbf{D};\mathbf{c})$. In the *NEGE* model, the ijth element is equal to 1 if $D_{ij}=0$, and $c_1exp(-D_{ij}/c_2)$ if $D_{ij}>0$, whereas in the *GSS* model, it is equal to 1 if $D_{ij}=0$, and $c_1exp\left(-(D_{ij}/c_2)^2\right)$ if $D_{ij}>0$. The corresponding element of the *SPH* model is equal to 1 if $D_{ij}=0$, $c_1(1-3D_{ij}/2c_2+D_{ij}^3/2c_2^3)$ if $0<D_{ij}<c_2$, and 0 if $D_{ij}\geqq c_2$. The abbreviations *NEGE*, *GSS*, and *SPH* reflect the choices of the negative exponential, Gaussian, and spherical forms for the correlation function, respectively. Weight matrix models vary in accordance with the specification of the elements of the matrix from which the weight matrix is derived by row standardization. In the *NN* model, the ijth element is equal to 1 if $D_{ij}>0$ and the jth location is one of the c_2 locations nearest to the ith location, and 0 otherwise. The abbreviation *NN* reflects the property that the element is specified with nearest neighbors. In the

[4] When the context of the discussion leaves no ambiguity, the dimensions of vectors and matrices are not elucidated.

[5] In this setting, a shorthand device is adopted to represent the elements of $K(\mathbf{D};\mathbf{c})$.

[6] The dimensions of $\mathbf{0}$ and $\mathbf{I}$ vary according to the context of the discussion. It is noteworthy that if the prevailing definition is applied to the weight matrix, $W(\mathbf{D};c_2)$ can be replaced with $W(\mathbf{D})$. Dubin (2003) and Kato (2008a) demonstrated that the parametric definition is superior to the prevailing definition. For an empirical application of the parametric definition, see Pace and Gilley (1997). It is also noteworthy that the above definition of the weight matrix model is based on the *simultaneous* approach. As stated in Militino et al. (2004), the *conditional* approach is rarely adopted in the hedonic literature. For reasons for such rarity, see Anselin (1988).