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Irreversibility, mean reversion, and investment timing

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ABSTRACT

This paper examines the effect of irreversibility on investment under mean reversion. We develop a continuous-time model wherein a risk-neutral firm is endowed with a perpetual option to invest in a project at any time by incurring a fixed investment cost at that instant. The project, once undertaken, generates a stream of cash flows that are governed by a mean-reverting stochastic process. The firm is then allowed to liquidate its project at any time to partially recover the fixed investment cost. The recovery rate of the fixed investment cost inversely gauges the degree of irreversibility of investment. Using a real options approach, we derive an analytical solution to the value of the firm that is analogous with an American compound option. We show that greater irreversibility of investment induces the firm to raise its investment trigger, thereby deferring the undertaking of the project. We further show that greater irreversibility of investment has a detrimental effect that makes the firm less valuable.

1. Introduction

In a recent article in this journal, Wong (2010) has examined the effect of irreversibility on investment under uncertainty. To this end, Wong (2010) develops a continuous-time model wherein a risk-neutral firm possesses a perpetual option to invest in a project at any time by incurring an investment cost at that instant. The project, once undertaken, generates a stream of cash flows that are governed by a geometric Brownian motion. The firm is then allowed to liquidate the project at any time to partially recover the investment cost. The recovery rate of the investment cost inversely gauges the degree of irreversibility of investment. Two main results of Wong (2010) are in order.[1] First, greater irreversibility of investment induces the firm to raise the threshold cash flow (i.e., the investment trigger) at which the project is undertaken. Second, greater irreversibility of investment has a detrimental effect that makes the firm less valuable. These results are rather intuitive in that reversibility of investment provides insurance to the firm against downside risk. Limiting the scope to recover the investment cost reduces the insurance benefit. The firm as such is less willing to undertake the project and also is less valuable.

The purpose of this paper is to re-examine the findings of Wong (2010) by incorporating mean reversion into the project's stream of cash flows. There is ample empirical evidence that corporate earnings are mean-reverting (Fama and French, 2000; Freeman et al., 1982; Kormendi and Lipe, 1987; Sarkar and Zapatero, 2003; Titman and Tsyplakov, 2007). Furthermore, a geometric Brownian motion is unbounded above, making it inappropriate to describe an equilibrium output price process resulting from entry and exit of firms (Bhattacharya, 1978; Bessembinder et al., 1995; Chu and Wong, 2010; Lund, 1993; Metcalf and Hassett, 1995; Sarkar, 2003; Tsekrekos, 2010; Wong, 2009, 2011). As such, we adopt a mean-revering stochastic process, which includes a geometric Brownian motion with a non-positive drift as a special case, to govern the evolution of the project's cash flow over time.

We derive an analytical solution to the value of the firm, which contains the solution in Wong (2010) when the mean-revering stochastic process reduces to a geometric Brownian motion with a non-positive drift. We then show, both analytically and numerically, that greater irreversibility of investment induces the firm to raise its investment trigger and reduces the value of the firm. Hence, the results of Wong (2010) are robust to incorporating more general stochastic processes that exhibit mean reversion. Finally, we show that the firm never finds it optimal to liquidate the project should the investment be sufficiently irreversible, which is the case when the speed of mean reversion is high and/or the long-term mean cash flow is large. Hence, projects with persistent cash flows that revert to high long-term mean levels are unlikely to be abandoned or liquidated.

The rest of this paper is organized as follows. Section 2 delineates our continuous-time model of a risk-neutral firm that has a perpetual option to invest in a project under uncertainty, where the investment timing is within the firm's discretion. After the investment option has been exercised, the firm possesses a perpetual option to liquidate the project for its salvage value. The liquidation decision, once made, is irreversible. Section 3 derives the value of the firm after the investment option has been exercised, while the liquidation option is still

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[1] Another interesting result of Wong (2010) is that the optimal investment intensity is invariant to the degree of irreversibility. However, Wong (2012) shows that such a neutrality result no longer holds when there are operating costs.

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alive. Section 4 goes back to derive the value of the firm before the investment option is exercised. Section 5 performs the comparative static analysis that examines how changes in irreversibility of investment affect the firm's investment trigger and the value of the firm. The final section concludes.

2. The model

Consider a risk-neutral firm that possesses a perpetual option to invest in a project.[2] Time is continuous and indexed by $t \in [0, \infty)$. The riskless rate of interest is constant at $r > 0$ per unit time.

Following Chu and Wong (2010), Sarkar (2003), and Tsekrekos (2010), we assume that the project's cash flow, X_t, at time t is governed by the following mean-reverting stochastic process:

$$dX_t = \lambda(\bar{X} - X_t)\, dt + \sigma X_t dZ_t, \tag{1}$$

where $\lambda \geq 0$ measures the speed of mean reversion, $\bar{X} \geq 0$ is the long-term mean cash flow to which X_t reverts, $\sigma > 0$ is the constant standard derivation of the rate of change of cash flows, and dZ_t is the increment of a standard Wiener process under the risk-neutral probability space, $(\Omega, \mathcal{F}, \mathcal{Q})$.[3] Mean reversion in the cash flow process captures the idea that cash flows over time have the tendency to revert to a normal long-run level, albeit subject to good or bad idiosyncratic shocks. It is worth mentioning that Eq. (1) reduces to describe a geometric Brownian motion with zero drift when $\lambda = 0$, and a geometric Brownian motion with a drift of $-\lambda$ when $\bar{X} = 0$.

The firm can undertake the project at any time by incurring a fixed investment cost, $I > 0$, at that instant. It is well-known (see, e.g., Dixit and Pindyck, 1994) that finding the optimal time to exercise the investment option is tantamount to finding a sufficiently large threshold value, X_I, of the state variable, X_t, such that the firm optimally invests in the project at the first instant when X_t reaches X_I from below. We refer to X_I as the firm's investment trigger.

After the investment has been made, the firm is allowed to liquidate the project at any future time, where the liquidation decision, once made, is irreversible. The salvage value of the project, net of any closing costs, is equal to $(1-s)I$, where $s \in [0,1]$ denotes the fraction of the fixed investment cost, I, that is lost upon liquidation.[4] A higher value of s implies a higher degree of irreversibility of investment such that $s = 0$ and $s = 1$ signify full reversibility and complete irreversibility, respectively. It is well-known (see, e.g., Dixit and Pindyck, 1994) that finding the optimal liquidation instant is tantamount to finding a sufficiently small threshold value, X_L, of the state variable, X_t, such that the firm optimally exercises the liquidation option at the first instant when X_t reaches X_L from above. We refer to X_L as the firm's liquidation trigger.

The firm's decision problems are solved by using backward induction. In Section 3, we derive the value of the firm, $V(X)$, after the project has been undertaken, where X is the current value of the state variable. In this case, the firm possesses the liquidation option that can be viewed as a perpetual American put option with an exercise price set equal to the salvage value, $(1-s)I$, where the underlying asset is the project's subsequent cash flows. The firm's liquidation trigger, X_L, is endogenously determined such that the value of the liquidation option is maximized. In Section 4, we derive the value of the firm, $F(X)$, before the investment is made, where X is the current value of the state variable. In this case, the value of the firm is equal to the value of the investment option that can be viewed as a perpetual American compound call option with an exercise price set equal to the fixed investment cost, I, where the underlying assets consist of the project's stream of cash flows and the liquidation option. The firm's investment trigger, X_I, is endogenously determined such that the value of the investment option is maximized.

3. Firm value after investment

Applying Ito's lemma, the instantaneous expected return on the firm's cash flow plus liquidation option is given by

$$\mathrm{E}\left[\frac{dV(X)}{dt}\right] + X = \frac{1}{2}\sigma^2 X^2 V''(X) + \lambda(\bar{X} - X)V'(X) + X, \tag{2}$$

for all $X \geq X_L$, where $\mathrm{E}(\cdot)$ is the expectations operator with respect to the risk-neutral measure, $\mathcal{Q}$. To preclude arbitrage, the expected return given by Eq. (2) must equal the riskless return, $rV(X)$, thereby yielding the following second-order ordinary differential equation:

$$\frac{1}{2}\sigma^2 X^2 V''(X) + \lambda(\bar{X} - X)V'(X) - rV(X) + X = 0, \tag{3}$$

for all $X \geq X_L$. It is easily verified that

$$V^0(X) = \left(\frac{1}{r+\lambda}\right)\left(\frac{X + \lambda\bar{X}}{r}\right), \tag{4}$$

is a particular solution to Eq. (3). Indeed, $V^0(X)$ is the present value of all subsequent cash flows generated by the project should the firm be prohibited from liquidation.

Define the following constant[5]:

$$\alpha = \sqrt{\left(\frac{1}{2} + \frac{\lambda}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} - \frac{1}{2} - \frac{\lambda}{\sigma^2} > 0, \tag{5}$$

and the following confluent hypergeometric function (Abramowitz and Stegun, 1972):

$$M_1(X) = 1 + \left(\frac{\alpha}{2 + 2\alpha + 2\lambda/\sigma^2}\right)\frac{2\lambda\bar{X}}{\sigma^2 X} + \left(\frac{\alpha}{2 + 2\alpha + 2\lambda/\sigma^2}\right)\left(\frac{\alpha + 1}{2 + 2\alpha + 2\lambda/\sigma^2 + 1}\right)\left(\frac{2\lambda\bar{X}}{\sigma^2 X}\right)^2 \frac{1}{2!} + \cdots, \tag{6}$$

for all $X > 0$. Let $\eta_1(X) = M'_1(X)X/M_1(X)$ be the elasticity of $M_1(X)$. It is evident that $M_1(X) = 1$ and $\eta_1(X) = 0$ if $\lambda\bar{X} = 0$, i.e., if the state variable, X, follows a geometric Brownian motion with a non-positive drift.

The value of the firm, $V(X)$, is the sum of the particular solution, Eq. (4), and the solution to the homogeneous part of Eq. (3) that captures the value of the liquidation option, which is a put option and thus attracts the constant, α. $V(X)$ must satisfy the following three boundary conditions:

$$\lim_{X \to \infty} \frac{V(X)}{X} < \infty, \tag{7}$$

$$V(X_L) = (1-s)I, \tag{8}$$

[2] The assumption of risk neutrality is innocuous as long as there are arbitrage-free and complete financial markets in which assets can be traded to span the state variable that determines the value of the firm.

[3] Chang and Chen (2012), Dixit and Pindyck (1994), Metcalf and Hassett (1995), Pindyck (1991), and Wong (2011) use an alternative process, known as the stochastic logistic or Pearl–Verhulst differential equation in population biology, to describe the evolution of the project's cash flows. Tsekrekos (2010) convincingly argues that Eq. (1) is more plausible, in particular for the description of an equilibrium output price process, due to the homogeneity property that Eq. (1) is homogeneous of degree one of the pair, $(\bar{X}, X)$.

[4] Following Wong (2010), we specify the salvage value of the project by using the exogenously given parameter, s. As suggested by an anonymous referee, it is of great interest to endogenize the determination of s. We leave this challenge for future research.

[5] Note that $-\alpha$ is the negative root of the fundamental quadratic equation, $\frac{1}{2}\sigma^2 y(y-1) - \lambda y - r = 0$.