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An improved inventory model with partial backlogging, time varying deterioration and stock-dependent demand

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ABSTRACT

This paper expands an inventory model for deteriorating items with stock-dependent demand. This model provides time varying backlogging rate as well as time varying deterioration rate. The aim of this model is to determine the optimal cycle length of each product such that the expected total cost (holding, shortage, ordering, deterioration and opportunity cost) is minimized. Further, the necessary and sufficient conditions are provided to show the existence and uniqueness of the optimal solution. Lastly, some numerical examples, sensitivity analysis along with graphical representations are shown to illustrate the practical application of the proposed model.

1. Introduction

In recent years, many researchers/scientists have discussed on inventory models for deteriorating items. In daily life, the deterioration of items becomes a common factor. Generally, deterioration indicates the damages of the products. To highlight such type of phenomenon, Ghare and Schrader (1963) first developed a deteriorating inventory model with constant rate of decay. That model was again extended by Covert and Philip (1973) with two-parameter Weibull distribution. Dave and Patel (1981) derived an EOQ (economic order quantity) model for deteriorating items with time-proportional demand and without shortage. Sachan (1984) extended the model of Dave and Patel (1981) by allowing shortages. Later, several related articles were discussed by Wee (1995), Hariga (1996), Wee and Law (1999), Moon et al. (2005), Chung and Wee (2007), Sana (2010a), Widyadana et al. (2011), and others. Widyadana and Wee (2011) addressed EOQ models for deteriorating items with different increasing demands. Sarkar et al. (2013) presented an EOQ model for deteriorating items with finite production rate and time dependent increasing demand.

In most of the above referred paper, the authors considered constant deterioration rate. But, in real life situation, items may deteriorate due to expiration of their maximum life time i.e., deterioration rate is proportional with time and the maximum life time can be controlled by the production system, i.e., the manufacturer can fix the maximum life time of the product. An inventory model with time dependent deterioration rate, shortage and ramp-type demand rate was discussed by Giri et al. (2003). Manna and Chaudhuri (2006) presented an EOQ model for deteriorating items for both time-dependent demand and deterioration. Loa et al. (2007) represented an integrated production-inventory model for imperfect production with Weibull distribution deterioration under inflation. An inventory model with general ramp type demand rate, Weibull deterioration rate and partial backlogging of unsatisfied demand was considered by Skouri et al. (2009). Sana (2010b) discussed an EOQ model with time varying deterioration and partial backlogging rate. In this model, the deterioration function was considered as functional form of time. Sarkar (2012a) developed an EOQ model for finite replenishment rate where demand and deterioration rate were both time-dependent. Sett et al. (2012) discussed a two-warehouse inventory model with quadratically increasing demand and time varying deterioration. Most recently, Sarkar (2012b) developed a production-inventory model for three different types of continuously distributed deterioration functions. In that paper, Sarkar (2012b) solved the model with the help of algebraical operation and a comparison between the different probabilistic deterioration models was shown by numerical experiments.

In the classical economic order quantity model, it is often assumed that the shortages are either completely backlogged or completely lost. In reality, often some customers are willing to wait until replenishment, especially if the wait will be short, while others are more impatient and go elsewhere. To reflect this phenomenon, Padmanabhan and Vrat (1995) considered an EOQ model for perishable items with stock-dependent demand under instantaneous replenishment with zero lead time. Abad (1996) discussed a pricing and lot-sizing problem for a product with a variable rate of deterioration by allowing shortages and partial backlogging. Chang and Dye (1999) recently developed an inventory model in which the backlogging rate was the reciprocal of a linear function of the waiting time. Cárdenas-Barrón (2001) presented an inventory model with shortage by an algebraical approach. An optimal replenishment policy for non-instantaneous deteriorating items

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with time varying partial backlogging rate was presented by Wu et al. (2006). Cárdenas-Barrón (2007) presented an inventory model on optimal manufacturing batch size with rework process at single-stage production system. A simple derivation on optimal manufacturing batch size with rework process at single-stage production system was presented by Cárdenas-Barrón (2008). An economic production quantity (EPQ) model with rework process at a single-stage manufacturing system with planned backorders was presented by Cárdenas-Barrón (2009). Most recently, Cárdenas-Barrón (2010, 2011, 2012), Roy et al. (2011a, b), Wee and Wang (2012), Yang et al. (2010) done their excellent research in this direction.

In the marketing management policy, display stock level plays a very important role in different sectors. Thus, it is very clear that the demand rate increased rapidly if the stored amount is high and vice-versa. Liao et al. (2000) were the first to discuss an inventory model for initial-stock-dependent consumption rate with permissible delay in payment. Dye and Ouyang (2005) extended Padmanabhan and Vrat's (1995) model with linear time-proportional backlogging rate, and then established the unique optimal solution to the problem for non-profitable building up inventory. Alfares (2007) found out an inventory model with stock-level dependent demand rate and variable holding cost. Chung and Wee (2007) developed the scheduling and replenishment plan for an integrated deteriorating inventory model with stock-dependent selling rate. Sana and Chaudhuri (2008) presented a deterministic EOQ model with stock-dependent demand rate where a supplier gives a retailer both a credit period and a price discount on the purchase of merchandise. In this direction, some notable researches were addressed by Goyal and Chang (2009), Roy et al. (2010, 2011c), Sana (2011a,b,c, 2012a), Sarkar et al. (2010a,b), and others. Sana (2012b) studied a newsboy problem with price-dependent demand and stochastic selling price to maximized the total profit. An EOQ model for perishable item with stock-dependent demand and price discount rate was presented by Sana (2012c). Sarkar (2012c) developed an EOQ model with finite replenishment rate to investigate the retailer's optimal replenishment policy under permissible delay in payment with stock dependent demand. See Table 1 for comparisons.

In the proposed paper, we develop an inventory model for time varying deteriorating items with stock dependent demand, shortage and partial backlogging. To the authors' knowledge, this type of model has not yet been considered by any of the researchers/scientists in inventory literature. Therefore, this model has a new managerial insight that helps a manufacturing system/industry to gain maximum profit. The rest of the paper is designed as follows: In Section 2, fundamental notation and assumptions are given. In Section 3, mathematical model is shown. Solution procedure of the model is provided in Section 4. Numerical examples and sensitivity are given in Sections 5 and 5.1 respectively. In Section 6, we provide some special cases and their comparison. Finally, conclusions are made in Section 7.

2. Notation and assumptions

The following notation and assumptions are considered to develop the model:

2.1. Notation

$\phi(t)$	time dependent deterioration rate;
$D(I(t))$	stock dependent demand;
A	ordering cost (\$/order);
C_h	inventory holding cost (\$/unit/week);
C_d	deterioration cost (\$/unit/week);
C_s	shortage cost (\$/unit/week);
R	lost sale cost (\$/unit/week);
T	length of order cycle (week);
L	maximum life time of product (week);
Q	order quantity per cycle (units/week);
$TC(t_1,T)$	total relevant cost (\$/unit).

2.2. Assumptions

1 The deterioration function $\phi(t)$ depends on time as $\phi(t)=\gamma t$, where γ is a constant ($0<\gamma\le 1, t\ge 0$).
2 Within the time interval $[0,t_d]$, the product has no deterioration. Deterioration occurs within the time interval $[t_d,t_1]$ at a variable deterioration rate $\phi(t)$.
3 $I_1(t)$ denotes the inventory level at any time $t\in[0,t_d]$ without the deterioration of product. $I_2(t)$ stands for the inventory level at any time $t\in[t_d,t_1]$ with the product deterioration. $I_3(t)$ signifies the inventory level at any time $t\in[t_1,T]$ with the product shortage.
4 The demand rate $D(I(t))$ is known as a function of instantaneous stock level $I(t)$; $D(I(t))$ is taken as following form:

$$D(I(t)) = \begin{cases} \alpha+\beta I(t) & \text{if } i(t)>0, \\ \alpha & \text{if } i(t)\le 0. \end{cases}$$

5 Shortages as well as backlogging are allowed. It is considered that only a fraction of demand is backlogged, we denote it $B(t)=\frac{1}{1+\varepsilon t}$, where t is the waiting time and $\varepsilon>0$ is a constant backlogging parameter.
6 Replenishment rate is infinite and lead time is negligible.
7 A single type of item is considered in this model.

3. Mathematical model

Initially, the inventory cycle starts with maximum stock-level I_{max} at $t=0$. The inventory level decreases during the time interval $[0,t_d]$ due to stock-dependent demand. Finally, inventory level falls at zero level during the time interval $[t_d,t_1]$ due to both stock-dependent demand and deterioration. The total process repeats itself after a scheduling time T. The total inventory system is shown in Fig. 1.

From Fig. 1, it is shown that during the time interval $[0,t_d]$, the inventory level decreases owing to stock-dependent demand rate. Hence, to signify the inventory system at any time t, the governing differential equation is given by

$$\frac{dI_1(t)}{dt} = -[\alpha+\beta I_1(t)], \quad 0\le t\le t_d \quad \text{with } I_1(0)=I_{max}. \tag{1}$$

Solving Eq. (1), we obtain the inventory level $I_1(t)$ as

$$I_1(t) = \left(I_{max}+\frac{\alpha}{\beta}\right)e^{-\beta t}-\frac{\alpha}{\beta}, \quad 0\le t\le t_d. \tag{2}$$

Due to stock-dependent demand as well as deterioration, the inventory level decreases during the interval $[t_d,t_1]$. Thus, the differential equation is to represent the inventory system

$$\frac{dI_2(t)}{dt}+\phi(t)I_2(t) = -[\alpha+\beta I_2(t)], \quad t_d\le t\le t_1$$
$$\text{ie.,}\frac{dI_2(t)}{dt}+\gamma t I_2(t) = -[\alpha+\beta I_2(t)], \quad \text{with} I_2(t_1)=0. \tag{3}$$