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Fuzzy order quantity inventory model with fuzzy shortage quantity and fuzzy promotional index

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ABSTRACT

The article deals with a backorder EOQ (*Economic Order Quantity*) model with promotional index for fuzzy decision variables. Here, a profit function is developed where the function itself is the function of *m*-th power of promotional index (PI) and the order quantity, shortage quantity and the PI are the decision variables. The demand rate is operationally related to PI variables and the model has been split into two types for the multiplication and addition operation. First the crisp profit function is optimized, letting it free from fuzzy decision variable. Yager (1981) ranking index method is utilized here to have a best inventory policy for the fuzzy model. Finally, a graphical presentation of numerical illustrations and sensitivity analysis are done to justify the general model.

1. Introduction

The EOQ model is an essential methodology to overcome some bottlenecks of the supply chain (Cárdenas-Barrón, 2007; Cárdenas-Barrón et al., 2011, 2012a,b,c). Among many factors, promotional strategy is a more applicable issue in today's business strategy. The promotional effort is an important management strategy to introduce a new product to the customers when it is launched in the market. The promotional efforts are free gift, discount offer, delivery facilities, better services and advertising, etc. Nowadays these strategies are applied for all types of commodities, not only for new products. Goyal and Gunasekaran (1995) developed a production-inventory model while demand of the end customers is influenced by advertising. Krishnan et al. (2004) found out optimal promotional strategy to maximize the profit function. Szmerekovsky and Zhang (2009) developed two-layer supply chain model for retail price and advertising sensitive demand. Xie and Wei (2009) and Xie and Neyret (2009) investigated two-layer supply chain model to obtain optimal cooperative advertising strategies and equilibrium pricing of the chain. Recently, the works (Sana, 2010, 2011a,b, 2012; Sana and Chaudhuri, 2008) in this line are worth mentioning.

The classical backorder inventory model lacks a new kind of variable named promotional Index (PI).This index usually enhance the demand of the customers of any kind of commodities. However, the demand rate is a function of the PI variable. Since the order quantity and shortage quantity are functionally related to demand rate. Hence, they are functionally related to the PI variable alone. Generally speaking, in the competitive world, no variable is fixed and hence they are flexible in nature. Several research papers have been published in fuzzy environment. Vojosevic et al. (1996) fuzzified the order cost into trapezoidal fuzzy number in the backorder model. Using this propositions other authors We and Yao (2003) studied a fuzzy inventory with backorder for fuzzy order quantity and fuzzy shortage quantity. With the help of fuzzy extension principle, an economic order quantity in fuzzy sense for inventory without backorder model has been developed by Lee and Yao (1999). Yao et al. (2000) analyzed a fuzzy model without backorder for fuzzy order quantity and fuzzy demand quantity. A lot-size reorder point inventory model with fuzzy demands has been developed by Kao and Hsu (2002) considering the α-cut of the fuzzy numbers and they have used ranking index to solve the model. De and Goswami (2001) developed an EPQ model for decaying items considering fuzzy deterioration and constant demand rate. Author like De et al. (2003) developed an EPQ model for fuzzy demand rate and fuzzy deterioration rate using the α-cut of the membership function of the fuzzy parameters. De et al. (2008) studied an economic ordering policy of deteriorated items with shortage and fuzzy cost coefficients for vendor and buyer. Recently Kumar et al. (2012) developed a fuzzy model with ramp type demand rate and partial backlogging.

In this paper, we have used PI as the fuzzy variable. However, in practice, any decision maker may face lead time delay or uneven traffic situations for which an uncertain flexible shortage quantity occurs during backorder period that affects the total order quantity as a whole. Therefore instead of crisp rather we assume all the decision variables as fuzzy variables. As the best of our knowledge, no research papers were published along this direction. First we have

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optimized the profit function under crisp environment then we have constructed an optimized function in terms of PI variable in two types of operation namely, '×' and '+' naming Model-I and Model-II respectively. Here we have seen that the crisp optimal solution and the solution for optimized function of PI variable are same. The models have been solved for four different positions of the PI variables with respect to the crisp PI optimum. Using the lower and upper bounds of the α-cuts of the membership functions for the fuzzy variables, we have constructed ranking index for each of the variables and finally got a numerical result with some programming language. A sensitivity analysis and graphical illustrations have been done to justify the model.

The rest of the paper is organized as follows: the notations and assumptions are given in Section 2, Section 3 formulates the model, numerical examples are provided in Section 4, sensitivity analysis has been done in Section 5, Section 6 concludes the achievements of our proposed model.

2. Notations and assumptions

The following notations and assumptions are used to develop the model.

Notation

q	The order quantity per cycle.
D	Demand rate per year.
s	Shortage quantity per cycle
c_1	Setup cost per cycle (\$).
c_2	Inventory holding cost per unit quantity per cycle (\$).
c_3	Shortage cost per unit quantity per cycle (\$).
s_1	Selling price per unit item (\$)
p_1	Purchasing cost of unit item(\$).
k	Scale parameter of promotional cost.
m	A positive integer, elasticity parameter of promotional cost.
t_1	Inventory run time (years).
t_2	Shortage period (years)
T	Cycle time in years.
Z	Average profit of the inventory (\$).

Assumptions.

1. Replenishments are instantaneous.
2. The time horizon is infinite (years).
3. Shortages are permitted and backlogged.
4. Demand rate is publicity index dependent where $D = x * d(\rho)$; * denotes the '+'(addition) and '×'(multiplication) compositions operator and $d(\rho) = \frac{\tau\rho}{1+\rho}$, τ is a positive constant.
5. The linear membership function for fuzzy effort $\tilde{\rho}$ is given by

$$\mu_{\tilde{\rho}}(\tilde{\rho}) = \begin{cases} \frac{\rho-\rho_1}{\rho_0-\rho_1}, for \ \rho_1 \le \rho \le \rho_0 \\ \frac{\rho_2-\rho}{\rho_2-\rho_0}, for \ \rho_0 \le \rho \le \rho_2 \\ 0 \quad , otherwise \end{cases}$$

3. Formulation of the model

3.1. Crisp model

In this model (Fig. 1), inventory starts with $(q-s)$ quantities and the inventory level reaches to zero at time t_1. Then, the shortage starts and continues for the period $[0,t_2]$. Therefore, the total length of the cycle is $T = t_1 + t_2 = D/q$. Now, the average profit function, considering revenue from selling, inventory cost, shortage cost and cost for promotional efforts, is

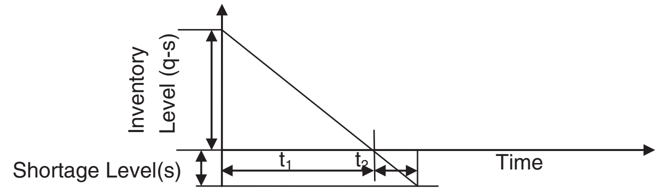

Fig. 1. Logistic diagram of inventory vs. time.

$$Z(q,s,\rho) = Revenue\, from\, selling\, items\, per\, unit\, time - setup\, cost\, per\, unit\, time - inventory\, cost\, per\, unit\, time - shortage\, cost\, per\, unit\, time - promotional\, cost\, per\, unit\, time$$

$$= (s_1 - p_1)D - \frac{c_1 D}{q} - \frac{c_2(q-s)^2}{2q} - \frac{c_3 s^2}{2q} - k\rho^m \tag{1}$$

where

$$\frac{q-s}{t_1} = \frac{s}{t_2} = \frac{q-s+s}{t_1+t_2} = \frac{q}{T} = D \tag{2}$$

Now, our objective is to maximize $Z(q,s,\rho)$ such that $q \ge 0$, $s \ge 0, \rho \ge 0$.

We shall discuss the model according to the nature of the demand rate:

3.1.1. Model-I

$$Maximize\ Z = (s_1 - p_1)D - \frac{c_1 D}{q} - \frac{c_2(q-s)^2}{2q} - \frac{c_3 s^2}{2q} - k\rho^m \tag{3}$$

where $D = \frac{x\tau\rho}{1+\rho}$ along with conditions in Eq. (2).

3.1.1. Model-II

$$Maximize\ Z = (s_1 - p_1)D - \frac{c_1 D}{q} - \frac{c_2(q-s)^2}{2q} - \frac{c_3 s^2}{2q} k\rho^m \tag{4}$$

where $D = x + \frac{\tau\rho}{1+\rho}$ along with conditions in Eq. (2).

Lemma. *For any values of* $\rho \ \varepsilon \ R+$, *the profit function* Z *in Eqs. (3) and (4) has unique maximum value at* $q^* = \sqrt{\frac{2c_1(c_2+c_3)D}{c_2 c_3}}$ and $s^* = \sqrt{\frac{2c_1 c_2 D}{c_3(c_2+c_3)}}$

Proof. Now, differentiating Z in Eqs. (3) and (4), we have

$$\frac{\partial Z}{\partial q} = \frac{1}{q^2}\left\{c_1 D + \frac{1}{2}(c_2+c_3)s^2\right\} - \frac{c_2}{2}, \frac{\partial Z}{\partial s} = c_2 - \frac{s}{q}(c_2+c_3),$$
$$\frac{\partial^2 Z}{\partial q \partial s} = \frac{\partial^2 Z}{\partial s \partial q} = (c_2+c_3)\frac{s}{q^2},$$
$$\frac{\partial^2 Z}{\partial q^2} = -\frac{1}{q^3}\left\{2c_1 D + (c_2+c_3)s^2\right\} < 0 \ \forall \ q \in R^+, s \in R^+, \rho \in R^+,$$
$$\frac{\partial^2 z}{\partial s^2} = -\frac{1}{q}(c_2+c_3) < 0 \ \forall q \in R^+.$$

For maximum values of Z, $\frac{\partial Z}{\partial q} = 0 = \frac{\partial Z}{\partial s}$ give us

$$q^* = \sqrt{\frac{2c_1(c_2+c_3)D}{c_2 c_3}} \ and \ s^* = \frac{c_2}{c_2+c_3}q^* = \sqrt{\frac{2c_1 c_2 D}{c_3(c_2+c_3)}} \tag{5}$$