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# Forward looking dynamics in spatial CGE modelling

Johannes Bröcker [a,b], Artem Korzhenevych [b,*]

[a] Department of Economics, University of Kiel, D-24098 Kiel, Germany
[b] Institute for Regional Research, University of Kiel, D-24098 Kiel, Germany

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ABSTRACT

This paper sets up a spatial dynamic CGE framework by combining the optimal growth model of saving and investment under adjustment costs and the spatial CGE model with Dixit–Stiglitz structure in the modern sector. Because of increasing product diversity on the dynamic equilibrium path, the model belongs to the category of semi-endogenous growth models. We overcome the difficulty of existing multiregional models to correctly approximate the infinite horizon equilibrium by employing a theoretically consistent terminal condition. The distinction of goods, factors, firms, and households by location, and the incorporation of trade costs in the model allow to study a variety of issues in regional and transport economics. We describe the model calibration and a tailor-made solution algorithm. The functionality is demonstrated using two illustrative examples.



## 1. Introduction

In recent years, spatial computable general equilibrium (SCGE) models have become a popular tool of regional impact analysis of policies, in particular in the area of transport economics (Bröcker and Mercenier, 2011; Tavasszy et al., 2011). The concept of an SCGE model implies the distinction of commodities, factors, firms, and households by location. This alone, however, is not the main distinguishing feature. Decisive for the ability of the SCGE models to bring forward new insights about the effects of policies is the incorporation of the fundamental principles of regional and spatial economics: factor mobility, economies of scale, and the presence of transport costs. These ideas are also central to modern trade and growth theory and to new economic geography (NEG). The corresponding theoretical framework is largely based on the work of Paul Krugman (Krugman, 1979, 1980, 1991).

A drawback of most existing SCGE models is that they are still static. Policies affecting the spatial distribution of economic activity, however, typically work with long delays, such that the dynamic transition period must not be neglected. Dynamic extensions are rare and "recursive" (e.g. Adams et al., 2000; Babiker et al., 2001; Giesecke and Madden, 2003; Ivanova, 2007), which means to concatenate static equilibria for each period by ad-hoc saving and investment functions. As saving and investment decision are not derived from neoclassical concepts, these models lack internal consistency. Furthermore, a proper welfare analysis of policy impacts is not possible within this framework, because intertemporal decisions are not made by optimizing agents.

Another family of applied models exhibits a kind of dynamic feature stemming from the NEG tradition (see e.g. Fan et al., 2000). In order to achieve a solution with profits of mobile firms equalized across space in such models one solves a sequence of static equilibria assuming firms to move from low profit to high profit locations between the steps of the sequence. Similarly, households move from low utility to high utility locations. In such a framework the subscripts indicating the individual steps of the sequence may be called time. But in fact they do not represent the movement of a real economy across time, because there is no saving and investment, and relocation decisions do not result from trading off future returns against present migration costs in a forward looking framework. The sequence has rather to be understood as an iteration for solving a static equilibrium with perfect mobility of households and firms. A truly dynamic perspective needed for policy analysis is however lacking there.

Multiregional forward-looking models are not too numerous due to substantial analytical and computational difficulties involved. A design fully consistent with the neoclassical basis of CGE modelling would require to derive saving as well as investment behavior from intertemporal optimization of households and firms in all locations. Furthermore, an appropriate solution method preserving the dynamic features and the theoretical consistency of the model must be designed. In particular, two important issues arise when operationalizing a forward-looking model with multiple optimizing agents (e.g. one per region): the approximation of the infinite horizon and asset ownership.

* Corresponding author at: DIW econ GmbH, Mohrenstraße 58, 10117 Berlin, Germany. Tel.: +49 30 20 60 972 21; fax: +49 30 20 60 972 99.
E-mail addresses: broecker@economics.uni-kiel.de (J. Bröcker), akorzhenevych@diw-econ.de (A. Korzhenevych).

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Two broad approaches to determine a finite-time alternative for the equilibrium conditions at infinity are compatible with a nonlinear solution strategy. The first is stemming from (Auerbach and Kotlikoff, 1987) and suggests fixing the terminal values of some of the variables at their steady-state values. As this requirement would have to bind after certain time, there is no assurance, however, that the system will thus converge to a point close to the steady state. The majority of models in the literature use this method (e.g. Devarajan and Go, 1998; Diao and Somwaru, 2000).

The second method is based on the use of the *local stable manifold theorem* of dynamic systems theory, presented, for example by (Irwin, 1980). The theorem says that, in general, what is true of the linearized system (in terms of determinacy and stability of equilibrium) is true of the original nonlinear system in some open neighborhood of the steady state. It is applied, for example, in Kehoe and Levine (1990) for the case of intergenerational interactions in a closed economy. A useful corollary of this theorem is that the stable subspace of the linearized system is the best affine approximation to the stable manifold of the nonlinear system around the steady state. Requiring a dynamic system to reach this stable subspace is therefore an instrument of choice for obtaining a precise approximation of the model dynamics.[1] The application of this theorem for the transformation of the boundary conditions at infinity in the CGE models is computationally more demanding than the first method. However, as we show, the use of modern solvers makes the exercise feasible.

Another major issue in a multiregional model that, however, is not given appropriate attention in the modelling literature is the ownership of capital. A precise approximation of the infinite horizon equilibrium requires the net asset positions of the households to be determined endogenously within the model (Lau et al., 2002). The existing multiregional models (McKibbin and Wilcoxen (1999), Bernstein et al. (1999), and Diao and Somwaru (2000), to name a few) do not possess this property, because imposing steady-state restrictions at an arbitrary time point in the future requires ad-hoc assumptions about the value of terminal assets. In contrast, in our approach we only have to specify the initial ownership shares of households in the regional capital stocks. The evolution of assets held by the households is then endogenously determined.

This paper sets up a dynamic spatial CGE framework by assuming households in every region to maximize a utility functional over time, taking their respective intertemporal budget constraints, prices and interest rates varying over time and space into account. Similarly, firms maximize present firm values. Households and firms are characterized by perfect foresight. Adjustment of capital stocks to shocks is smoothed by assuming the existence of adjustment costs for the capital stock. The specification of the production and household sectors as well as of the goods market is close to an earlier static model (Bröcker, 1998) which has been widely applied under the brand name *CGEurope* in transport policy evaluation (Bröcker et al., 2010; Korzhenevych and Bröcker, 2009). Like in the earlier model, we assume monopolistic competition in Dixit–Stiglitz style in the "modern sector". Because of increasing product diversity on the dynamic equilibrium path the model belongs to the category of semi-endogenous growth models in the sense of Jones (2005).

In addition to distinction of goods, factors, firms, and households by location, the spatial dimension in the model comes in through the costs for goods movement depending on geography. The total trade costs for goods to be delivered from one region to another are assumed to amount to a share in the traded value. The model is thus applicable for studying the spatial effects arising due to both, regional and transport policy measures. The way trade costs are modelled resembles – but is not identical with – the "iceberg" approach (Samuelson, 1954).

In the next section, we present all the steps of the model setup and the accompanying derivations. The resulting mathematical problem requires a tailor-made solution algorithm which we describe in Section 3. Section 4 studies the predictions of the model using an experimental 3-region setup. The question of numerical performance is addressed in Section 5. Section 6 concludes.

## 2. Model formulation

The model we are going to describe is a dynamic version of the earlier static model (Bröcker, 1998). Therefore, we will concentrate on the dynamic elements of the model and only shortly describe other parts. Agents of the economy are firms and households. The starting point is an open-economy version of the Ramsey optimal savings model, combined with the adjustment costs for investment framework (Abel and Blanchard, 1983). Thus both, households and firms make intertemporal decisions and have perfect foresight. As in the static model, the neoclassical structure is altered by the introduction of monopolistic competition in the "modern sector". The state is not modelled as an own sector.

The intertemporal problem is formulated in continuous time. All variables refer to one region and are functions of time. Real quantities are denoted by upper-case Latins, prices by lower-case Latins, and exogenous parameters by Greek letters. Exogenous parameters mostly do not have a regional index and are constant over time. Exceptions are explicitly mentioned. If not needed for understanding, the regional and time subscripts ($r$ and $t$) are omitted to avoid notational clutter.

### 2.1. Firms

Two types of goods are distinguished in the model: local and tradable. Local goods can only be sold within the region of production, while tradables are sold everywhere in the world (whereby trade costs arise), including the own region. Identical firms located in the region produce gross output $M$ by combining capital $K$, effective amount of labor service $E$, local goods $L$, and a CES composite $G$ of tradable goods coming from all regions, in a Cobb–Douglas (CD) technology,

$$M = \phi K^{\chi} E^{\theta} L^{\beta} G^{\gamma}, \tag{2.1}$$

with positive elasticities, where $\chi+\theta+\beta+\gamma=1$. The level of productivity $\phi_r$ may be different across regions, and no technological convergence is assumed. The regional population is assumed to be immobile and constant at $\bar{E}_r$. The effective amount of labor input is assumed to grow at an exogenous rate of technological progress, $\tilde{\xi}$, i.e.

$$E_r(t) = E_r e^{\tilde{\xi} t}. \tag{2.2}$$

Homogeneous gross output serves a double purpose: first, it is one-to-one transformed into the local good (without a price mark-up), and secondly, it is used as the only input in the production of a variety of tradable goods under increasing returns to scale. The market for local goods is perfectly competitive, while monopolistic competition with free entry in Dixit–Stiglitz style prevails in the tradables market. Each firm thus produces a different variety of a tradable good. The number of varieties supplied by the region is endogenously determined by the free entry condition, which means that all firms earn zero profit in equilibrium. By choice of units the mill price of tradable and local goods is the same and is denoted by $p$. A CES composite price of all tradables available in the region is denoted by $g$.

[1] In-between these two methods there is also an approach to impose a balanced growth constraint in the terminal period. It is applied e.g. in Bernstein et al. (1999) and Böhringer and Welsch (2004). Although applicable to a wider class of models than the first method, this approach however lacks the theoretical foundation provided by the local stable manifold theorem.